APPLICATION OF PRECAST CONCRETE ELEMENTS FOR ROOFS OF INDUSTRIAL BUILDING

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ABSTRACT

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APPLICATION OF PRECAST CONCRETE ELEMENTS

FOR ROOFS OF INDUSTRIAL BUILDING

Precast concrete elements are increasingly employed in various structures which are expected to meet the requirement of modern building industry. Basic provisions and requirements for design and construction of precast concrete are available. In this report, application of precast concrete elements for roofs of standard industrial building is discussed. An example of the application employing economical open-web joists and girders supporting large panels is presented.

ACKNOWLEDGEMENT

The writer wishes to express his profound gratitude to his supervisor, Dr. Zenon A. Zielinski, professor of Civil Engineering, Concordia University, for his valuable guidance, advice and encouragement during the course of his study.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (continued)

NOTATIONS

A = Average effective tension area of concrete

A_{ch} = Area of horizontal confinement reinforcement

A_{cv} = Area of vertical confinement reinforcement

A_s = Area of tension reinforcement

A_{sh} = Area of reinforcement for horizontal crack

A_{vf} = Area of shear friction reinforcement

C = Movement coefficient

F = Flexural coeficient

K_u = Strength coeficient of resistance

L = Span length centre to centre

L' = Clear span length

M = Bending moment

M_u = Ultimate resisting moment

N_u = Design tensile force on bracket or corbel acting simultaneously with V_u

P = Concentrated load

S = Short span length, two way slabs

V = Shear force

V_u = Total applied design shear force at section

Z = A quantity limiting distribution of flexural reinforcement

a = Depth of equivalent rectangular stress block

b = Width of compression face of flexural member

b_w = Width of web

c = Distance from extreme compression fibre to neutral axis at ultimate strength

d = Distance from extreme compression fibre to centroid of tension reinforcement

d_c = Distance from extreme tension fibre to the centre of the adjacent bar.

f'_c = Compressive strength of concrete

f_y = Yield strength of reinforcement

h_f = Depth of flange

m = Ratio of short span to long span, two-way slabs

n = Modulus ratio

s = Shear reinforcement spacing

v_{dh} = Design horizontal shear stress at section

v_u = Nominal total design shear stress

w = Uniformly distributed load

ρ = Ratio of area of tension reinforcement to effective area of concrete at section

ϕ = Capacity reduction factor

μ = Shear friction coeficient

CHAPTER 1

GENERAL INTRODUCTION

In the past twenty years, industrialization in the building industry has been proceeding at high speed all over the world especially in Europe and America. In European countries, the experiences reveal that industrialized building can reduce on site labour 30%, even up to 50%, and save one-third to one half of constructing time required by traditional building methods (1). This means that industrialized techniques are much more economical in terms of money and manpower, and buildings can be constructed at a much faster rate than with traditional methods. These advantages are obtained by the following(2).

(1) A high degree of mechanization

(2) Reduction of on-site labour

(3) Standardization of components and products

(4) Dimensional co-ordination

(5) Integration of building team - architect, engineer, fabricator, contractor and client.

(6) More sophisticated use of management techniques.

Standardization of constructive elements in repetitive fabrication, erection and assemblies to a high degree accelerates the industrialization and garantees the construction quality. It simplifies much the design and operation procedure. Standards must be adopted, particularly in dimensional co-ordination, to eliminate the useless waste in all branches of the industry. In fact, standardization is the basis and necessity for industrialization.

Panelization which has been experienced in recent years in many countries is a very important step along the road to industrialization. Prefabricated panels are commonly employed for walls, floors and roofs which are fabricated identically. Panelization which is widely adopted has the following advantages:

1) Panels can be very easily manufactured in a variety of kinds such as solid, cored, ribbed and sandwich panels.

2) Panels provide great design flexibility whether they can be used to produce a variety of box sizes or provide variations in room shapes and sizes.

3) Panels are easy to handle in shipping and erection stages and allow to avoid problems with highway traffic restrictions.

4) With panelized construction it is a simple matter to produce joints which provide adequate structural integrity for the building as a whole. Panelized buildings have survived Tashkent earthquakes.

Nowadays, precast concrete techniques are being used to an increasing extent in the repetitive production of modular, prefabricated and standardized building elements. These products are suitable for quick assembly of structural framework, walls, floors and roofs. Precast concrete can be moulded in many ways with a wide variety of surface treatments. It will play a very important part in the development of the industrialization. An even more significant contribution is that it can be simultaneously used, both structurally and architecturally, and also allows to have cast practically all of the required services. Within its structural framework, it is possible to fit in a compatible manner almost all other building products. This allows for greater versatility in architectural expression.

Generally precast concrete has the following main advantages:

1) Economical in formwork, shuttering and labour.
2) Fabrication independent upon weather conditions.
3) Appropriate methods of curing ensure improved and reliable products and accelerate maturing, and thus productivity is increased.
4) Better quality is ensured and sectional size can be reduced.
5) Speed of construction on site is greatly increased
6) Volumetric stability of precast structure is better since shrinkage has taken place before erection.

Prefabrication also allow to detain following structural qualities:

1) Low initial and maintanence cost
2) Long life and adequate structural capacity

3) Good insulating and accoustic characteristics.

4) Pleasing appearance

5) Weather resistance

According to different geographic conditions and the availability of materials and equipments, precast concrete elements should be designed reasonably in the appropriate methods to avoid undesirable waste and get the best result. This has been experienced in many countries and districts.

Universal Concrete Panel Building System (UCOPAN) has successfully experienced in many projects such as schools, universities, hospitals and low-cost housing in India, Nepal and other Asian countries where a series of panels of 1.5" thick concrete slab, reinforced with Wire Welded Fabric and stiffened with ribs, were employed for walls, floors and roofs. (Fig. 1.1 - Fig. 1.3) Panels were prefabricated in Modular sizes according to different projects depending on the space requirement and equipment available. These panelized buildings were designed and constructed under the consideration of the weather condition, material supplied, finance, and equipments, thus leading to the saving in the consumption of materials in terms of money and manpower.

From the above discussion, obviously, there is no doubt, that industrialization through standardization, panelization with precast techniques can bring a significant decrease in cost, construction time and manpower to the building industry.

In European countries, about 25% to 35% of housing construction is completed with precast systems. The Soviet Union predicts that by 1980 over 80% of its housing construction will be of precast systems.

Since the precast technique provides a view of great prospects in the future of building industry, application of precast concrete elements to structures with standardization and panelization to meet the requirements of industrialization is reasonable and desirable.

Fig. 1.1 (a)Roof and floor panels

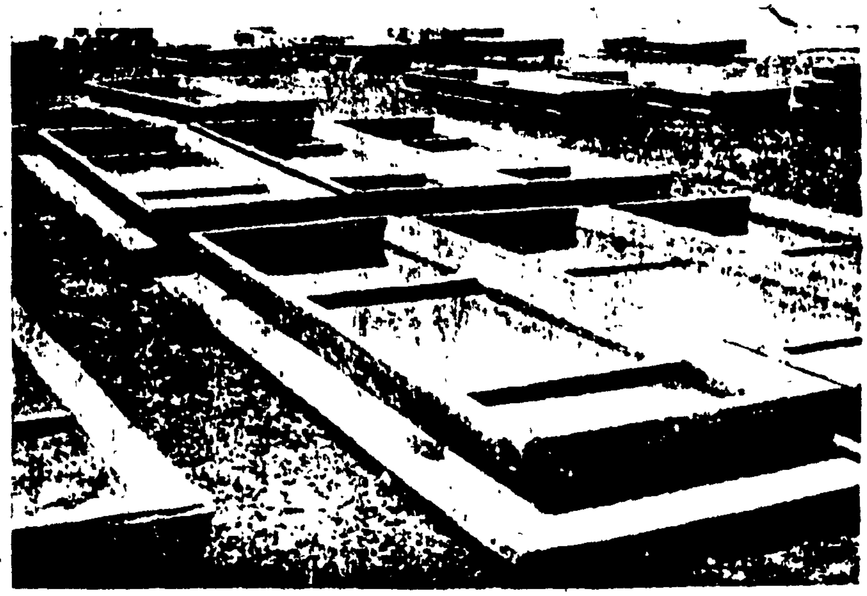


Fig.1.1(b) Wall panels

Fig. 1.1 UCOPAN Panels

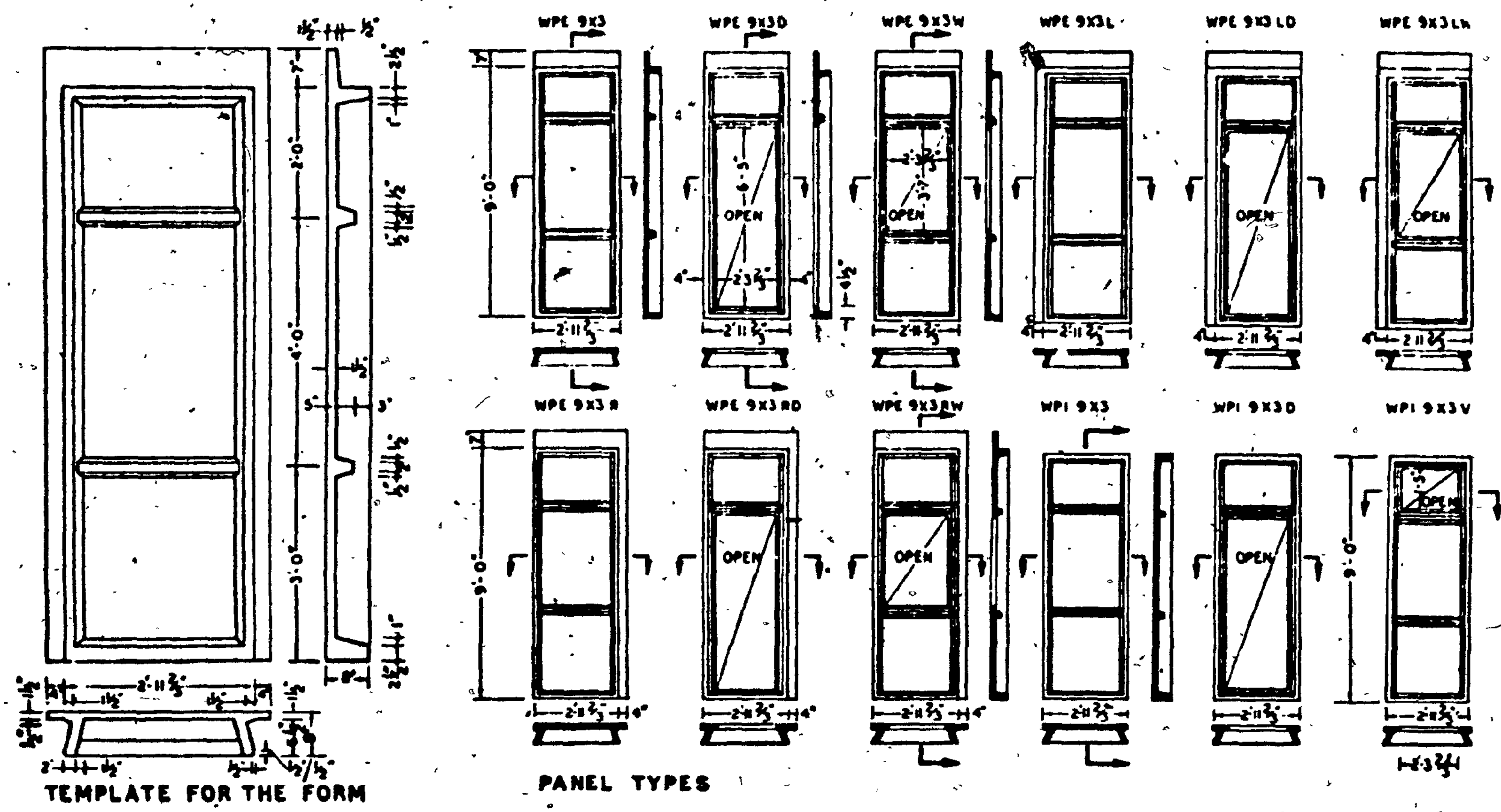


Fig. 1.2 Different types of panels produced in the same mould - UCOPAN

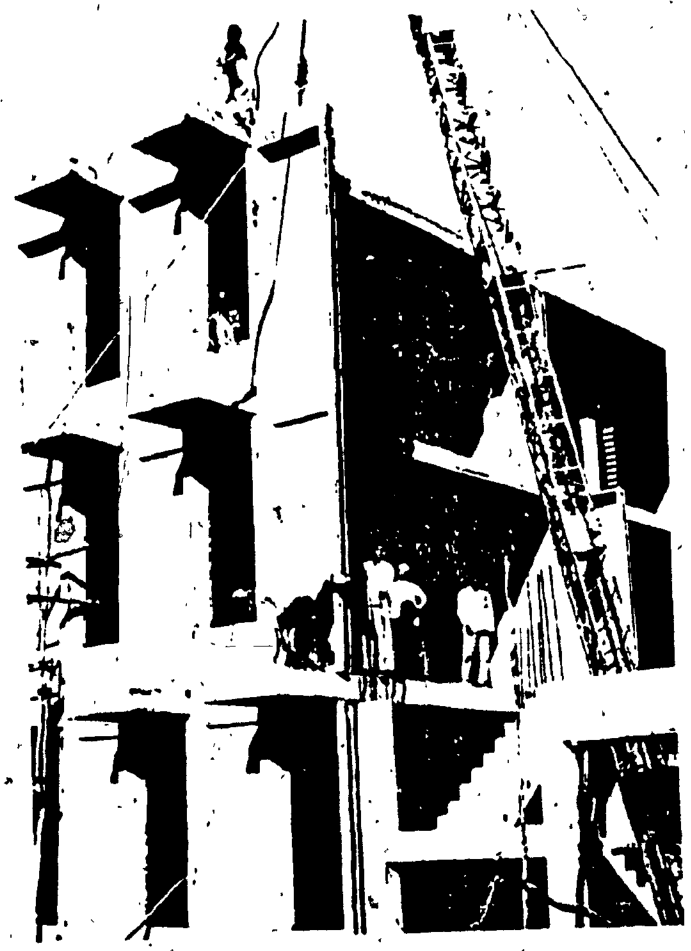


Fig. 1.3

Buildings assembled with standard panels - UCOPAN

CHAPTER 2

APPLICATION OF PRECAST CONCRETE IN INDUSTRIAL ROOFS

Recently, precast concrete elements for building roofs has been increasingly employed. Simplicity of construction is achieved with precast concrete techniques (Fig. 2.1 and Fig. 2.2).

A development of economic precast concrete roofs for industrial building is attempted here (Fig. 2.3 and Fig. 2.4).

A number of large span precast concrete panels of two way slabs reinforced with Welded Wire Fabric and stiffened with perimetric ribs are employed. Panels of large span eliminate unnecessary joints. The amount of labour in manufacture and erection is reduced, the construction time is shortened and the quality of internal finishes and utilities is higher.

The joists rest on the columns or brackets which are cast integrally at the mid-span of the girders. The brackets so cast ensure that the top edges of the joists and girders are at the same level which provides smooth continuity throughout the whole system as the composite section of joists and girders is being cast. The girders

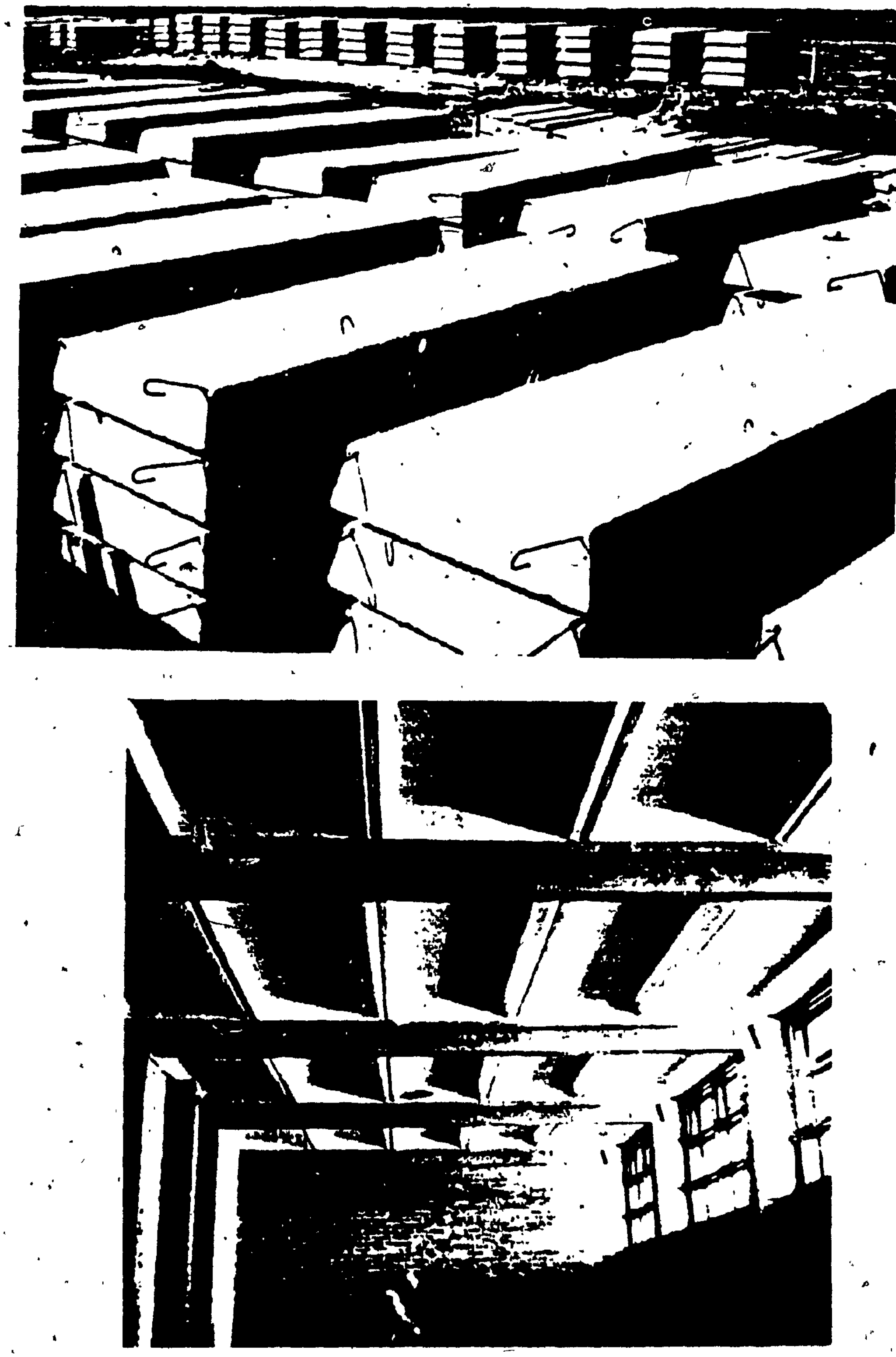


Fig. 2.1 Roof constructed with modular panels
supported by Tee girders - UCOPAN

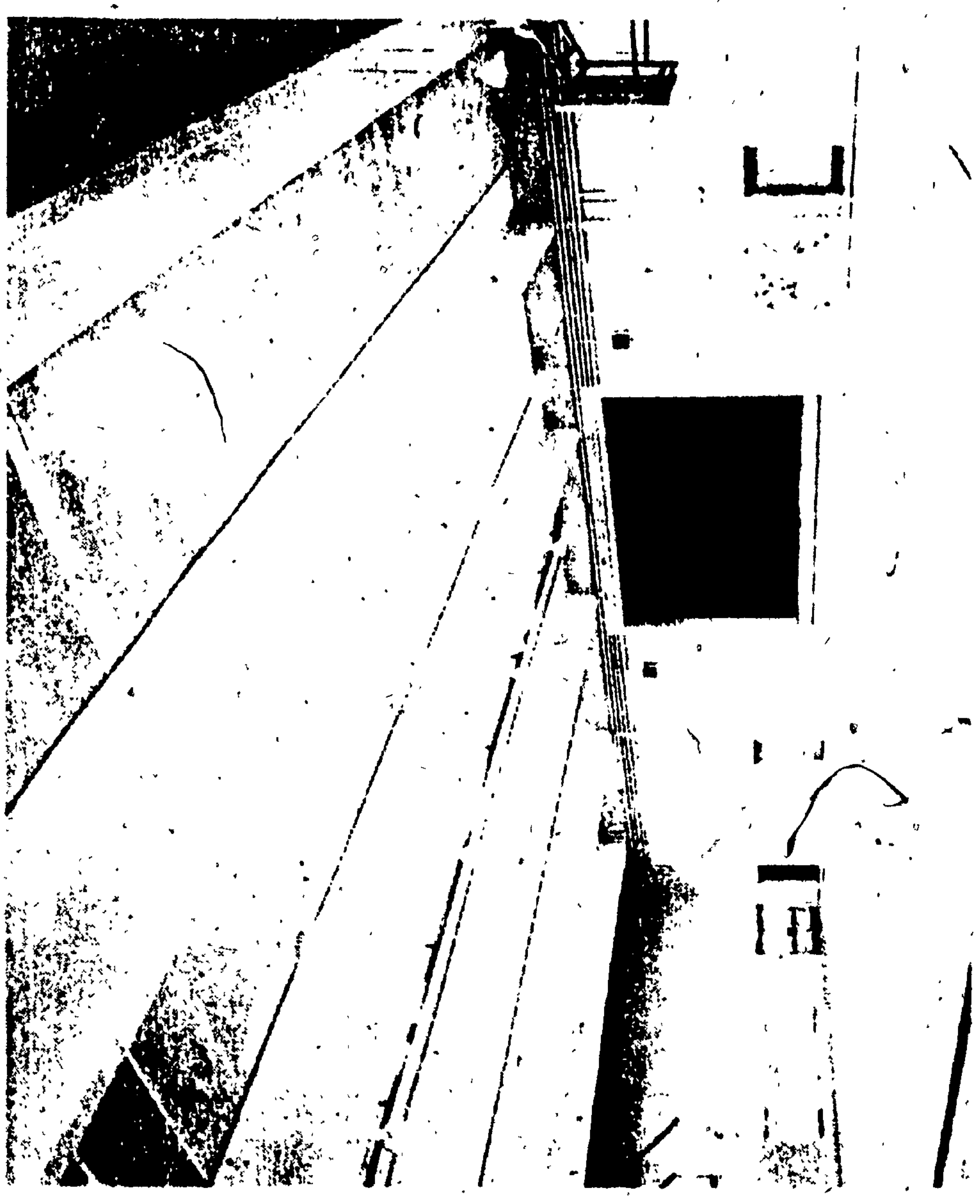


Fig. 2.2 Roof of single Tee supporting transverse panels (3)

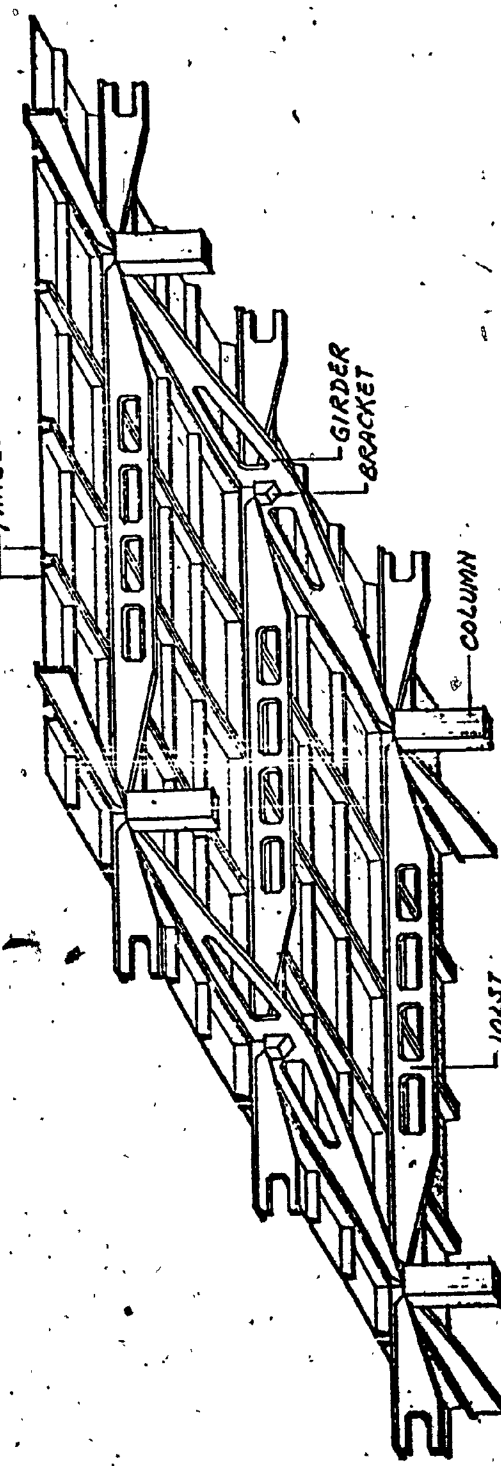


Fig. 2.3 Roof of precast concrete elements

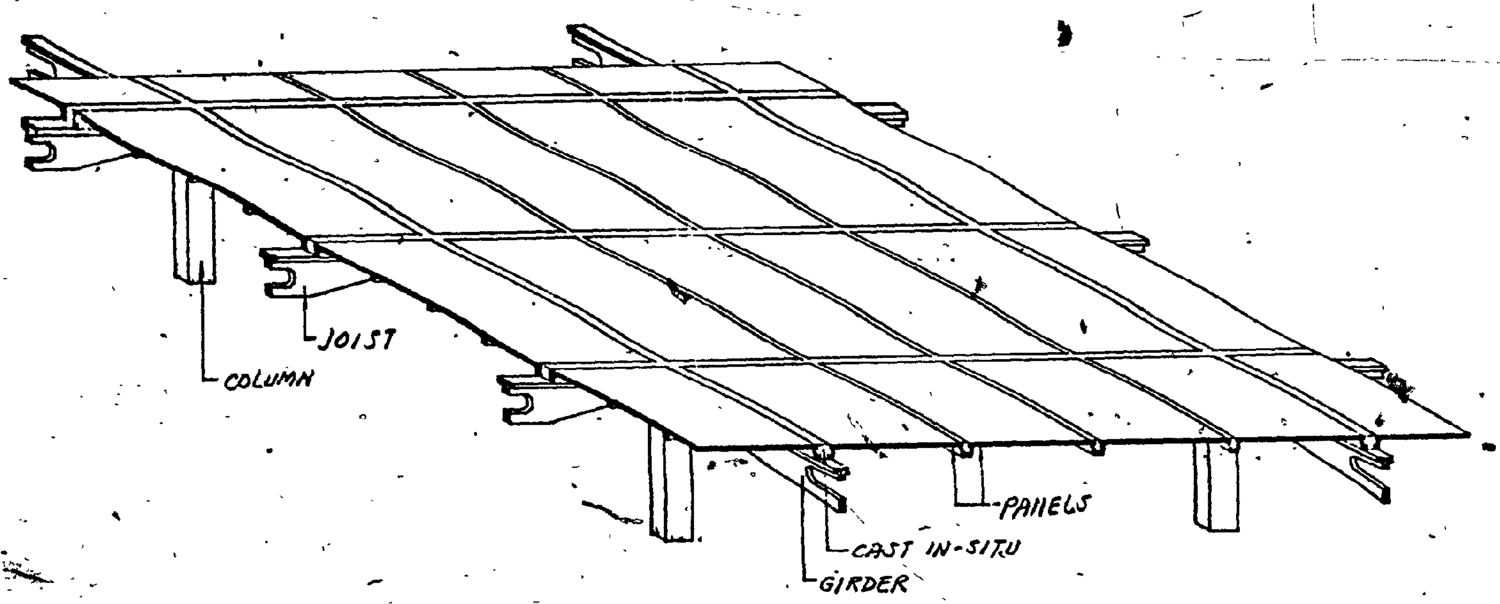


Fig. 2.4 Top view of roof of precast concrete elements

are supported directly on the columns.

The precast joists and girders are designed as simply supported beams in Tee Sections with economic open-webs which are designed on the basis of analysis and research tests and have been experienced in many projects in Europe and America. A series of standardized open-web girders of various spans had been developed in Poland from 1953-1957 [4]. The economical open-web girders replace the traditionally used solid full-web beams which at large spans become too heavy and with too much material being used.

Economic open-web girders represent a new structural system of minimum consumption of materials up to 50% saving of concrete and steel and resemble in shape to a catenary line or bending moment diagram. [5]

The composite portion is cast in-situ at the top of joists and girders. This portion minimizes the depth of the roof system and provides adequate connection between the panels to make the whole system act as a unit. Moreover the extended stirrups in the joists and girders provide adequate shearing connectors between the precast concrete and the in-situ concrete. The trapezoidal joist and girder lower their centres of gravity below the bearing level thus increasing the stability and eliminating overturning.

In practice, it is impossible to make products which will have exact design dimensions. In fact, extreme precision is pointless, small inaccuracies are unavoidable during erection. Hence tolerance should be provided for all precast concrete elements because the dimensions of precast concrete units are never exactly as theoretically specified.

In precast concrete structures, joints are the weakest points. Connection of precast components presents difficult technical probelms since the structure is only as strong as the joints. Connection design should consider the following:

(1) The load and actions to be resisted

(2) The structural function of the joint

(3) The fabrication and erection procedures.

In addition to gravity and lateral loads due to wind and earthquakes, the effect of volume changes due to shrinkage, creep and temperature, the effect of fabrication and construction tolerance error must be considered. Hence in common practice a large safety factor or load factor in design of connections should be employed.

In this system, both wet and dry connections are used. For wet connections, the perimeter ribs of panels are at an obtuse angle to the panel membrance and are shaped in such a way that when the panels are placed side by side,

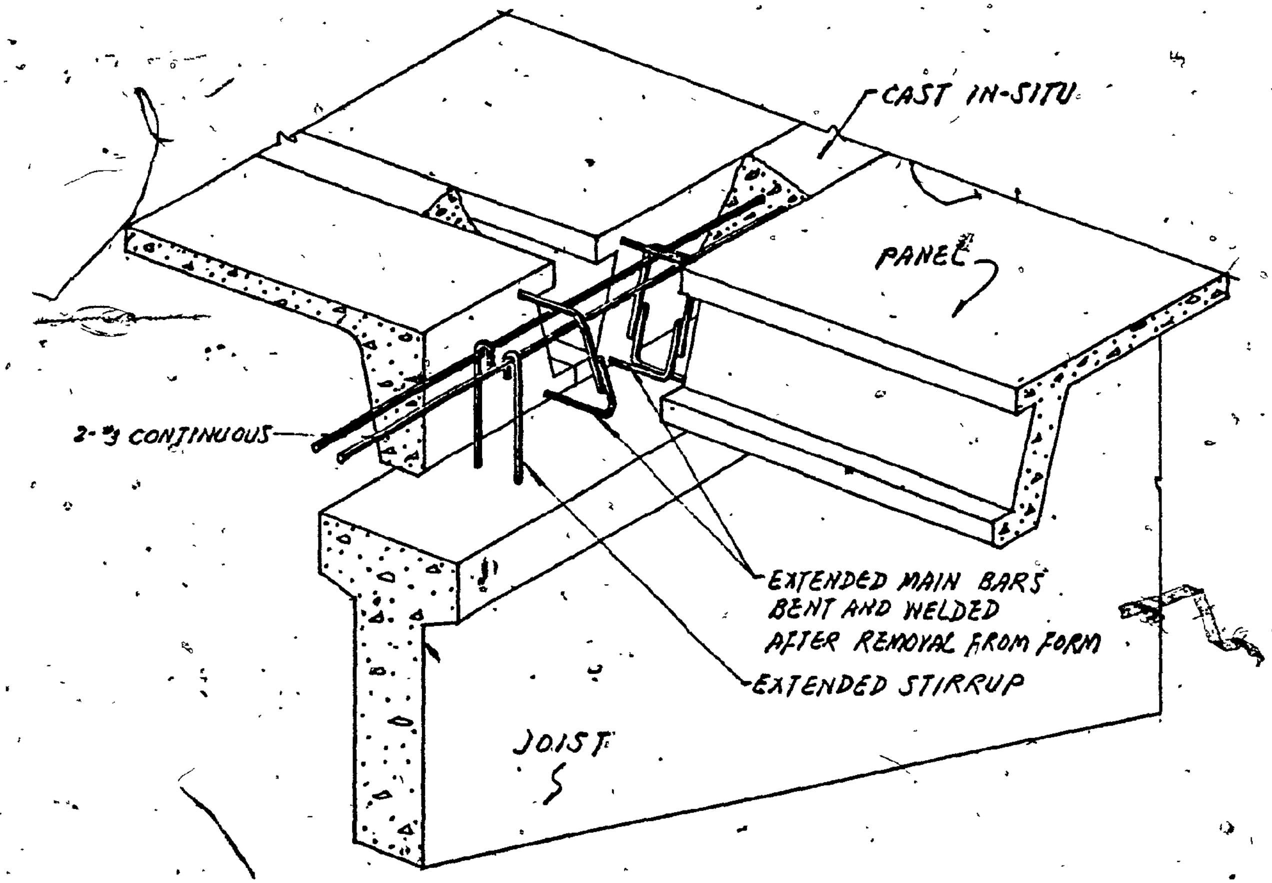


Fig. 2.5 Connection between the panels and joist

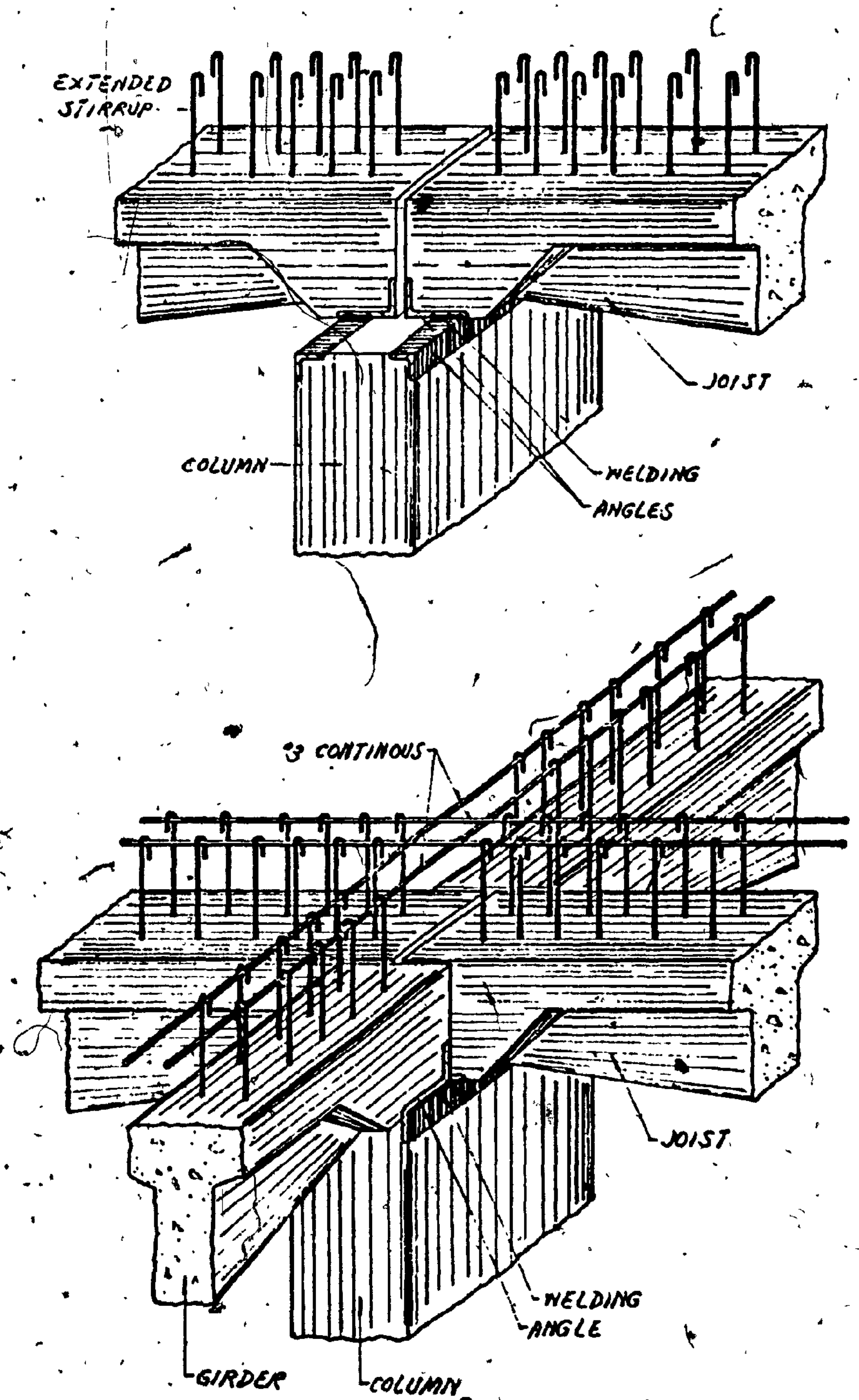


Fig.2.6 Connection between joists,girders and column

gaps exist between the ribs along the horizontal lines. The gaps can be filled in with additional reinforcement and concrete which tie separate panels into a solid and monolithic structure, durable and suitable for seismic zones and flood-prone areas. (Fig. 2.5)

The dry connection is completed by welding for the joints between the brackets and joists, the girders and the columns. These procedures are easily handled in every individual operation stage (Fig. 2.6).

This roof system provides plenty of headroom and space for the utility piping system such as lighting conduits, air condition ducts, and water supply pipes. Moreover, it provides convenience ~~for~~ for acoustic and heating insulation in installation processes.

CHAPTER 3

DESIGN PROCEDURE

The proposed roof system consists of panels, joists and girders, brackets, and connections. Each of these elements is designed following the practice of the American Concrete Institute (ACI), and Prestressed Concrete Institute as well as other references. (6) (7) (8) (9)

3.1 PANELS

The panels are designed as two-way slabs reinforced with wire welded fabric and supported by perimetric ribs.

The ACI Building Code (318-71) states that the slab should be designed by approved methods which shall take into account the effect of continuity and fixity at supports, the ratio of length to width of slab and the effect of two-way action. Hence in two-way slabs, the reinforcing steel is usually placed so that flexural resistance will be provided in two directions.

The ACI Code provides three separate approximate methods for use in determining shear and moments in slabs which distribute their load to the four edge supports. Traditionally, the procedure known as Method 2 has been used almost exclusively in engineering practice. This method has been devised considering the theory of elasticity and the results of experiments. The methods applies only when:(10)

a) The loads are uniformly distributed and

b) The ratio of live load to dead load does not exceed 3.

These recommendations are intended to apply to isolated or continuous slabs, supported on all four sides by ribs which are built monolithically with the slab.

The shear stress in the slab may be computed on the assumption that the load is distributed to the supports according to the equation: (10)

a) For short span, $w' = \frac{wS}{3}$ ....(i)

b) For long span, $w' = \frac{wS}{3} \times \frac{3 - m^2}{2}$ ....(ii)

The loads on the supporting ribs for a two-way rectangular slab may be assumed as that load contained within the tributary areas of the slab bounded by the intersection of 45° lines from the corners with the median line of the slab parallel to the long side.

The two-way slab shall be considered as consisting of strips in each direction as middle strip and column strip, the middle strip is one-half panel in width, symmetrical about the panel centre line and extending through the panel in the direction in which moments are considered. The width of the column strip is one-half panel in width occupying the two quarter-panel areas outside the middle strip.

The critical sections for moment calculations are referred to the principal design section and are located as follows:

a) For negative moment, along the edges of the panel at the forces of supporting ribs, and

b) For positive moment, along the centre line of the panels.

The bending moment for the middle strip shall be computed using the formula:

$$M = Cw'S^2$$

The coefficients C for different supporting conditions are listed in Table 1.

The average moments per foot of width in the column strip shall be two-thirds of the corresponding moment in the middle strip. Since the slab is reinforced with Welded Wire Fabric which is distributed equally through the spans in both directions, only the maximum positive and negative bending moments in middle strip shall be considered (on the safe side).

The ACI Code states that for monolithic or fully composite construction, the beams include that portion of the slab, each side of the beam extending a distance equal to the projection of the beam above or below the slab, which is greater, but not greater than four times the slab thickness. Two examples of the rule are provided in Fig. 3.1.

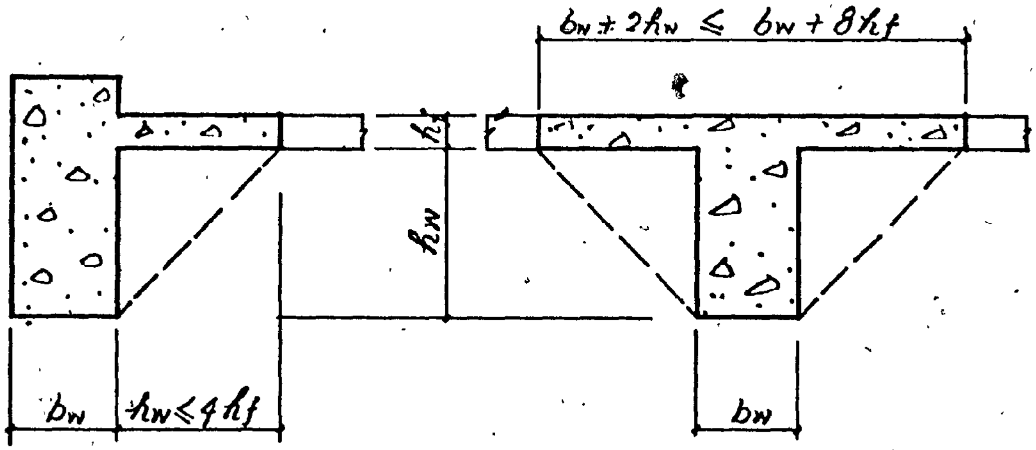


Fig. 3.1

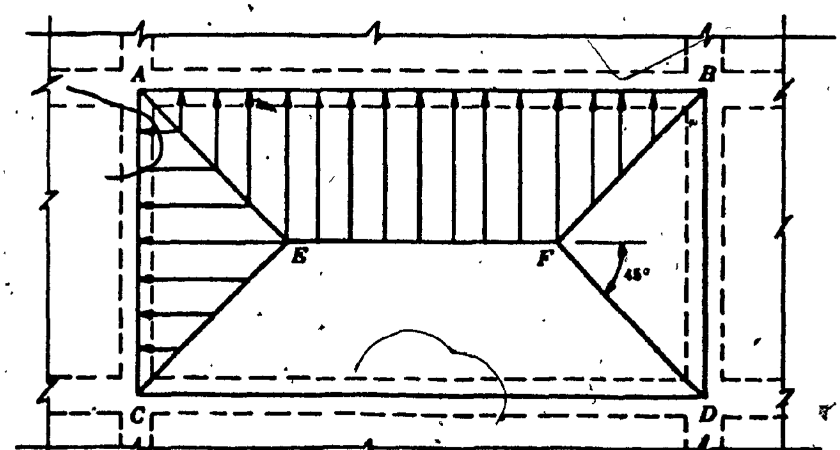
The bending moments in the perimeter ribs can be determined approximately by using an equivalent uniform load per liner foot of ribs for each slab supported as described in formulas (i) and (ii).

The longitudinal perimeter ribs are designed as continuous beams for live load by extending the main bars beyond both ends when precast. After the removal from forms the main bars shall be bent and welded. The ACI Code provides approximate coefficients of shear and bending moment which may be utilized when the following conditions are satisfied:

1) Adjacent clear spans may not differ in length by more than 20% of the shorter span.
2) The ratio of live load to dead load may not exceed 3.
3) The loads must be uniformly distributed.

TABLE I
Moment Coefficient For Two-way Slabs (10)

Moments	Short span						Long span, all values of m
	Values of m						
	1.0	0.9	0.8	0.7	0.6	0.5 and less	
Case 1 – Interior panels							
Negative moment at –							
Continuous edge	0.033	0.040	0.048	0.055	0.063	0.083	0.033
Discontinuous edge	—	—	—	—	—	—	—
Positive moment at midspan	0.025	0.030	0.036	0.041	0.047	0.062	0.025
Case 2 – One edge discontinuous							
Negative moment at –							
Continuous edge	0.041	0.048	0.055	0.062	0.069	0.085	0.041
Discontinuous edge	0.021	0.024	0.027	0.031	0.035	0.042	0.021
Positive moment at midspan	0.031	0.036	0.041	0.047	0.052	0.064	0.031
Case 3 – Two edges discontinuous							
Negative moment at –							
Continuous edge	0.049	0.057	0.064	0.071	0.078	0.090	0.049
Discontinuous edge	0.025	0.028	0.032	0.036	0.039	0.045	0.025
Positive moment at midspan	0.037	0.043	0.048	0.054	0.059	0.068	0.037
Case 4 – Three edges discontinuous							
Negative moment at –							
Continuous edge	0.058	0.066	0.074	0.082	0.090	0.098	0.058
Discontinuous edge	0.029	0.033	0.037	0.041	0.045	0.049	0.029
Positive moment at midspan	0.044	0.050	0.056	0.062	0.068	0.074	0.044
Case 5 – Four edges discontinuous							
Negative moment at –							
Continuous edge	—	—	—	—	—	—	—
Discontinuous edge	0.033	0.038	0.043	0.047	0.053	0.055	0.033
Positive moment at midspan	0.050	0.057	0.064	0.072	0.080	0.083	0.050



The load on element *ABEF* is supported by beam *AB*.
The load on element *AEC* is supported by beam *AC*.

The following approximate formulas from the ACI Code may be used to determined the shear forces and bending moments in continuous beams.

FOR POSITIVE MOMENT

End Spans:

If discontinuous end is unrestrained...$w(L')^2/11$

If discontinuous end is integral with the support ....$w(L')^2/14$

Interior spans...........................$w(L')^2/16$

FOR NEGATIVE MOMENT

Negative moment at exterior face of first interior support:

Two spans................................$w(L')^2/9$

More than two spans......................$w(L')^2/10$

Negative moment at other faces of interior supports..................................$w(L')^2/11$

Negative moment at face of all supports for

(a) slabs with spans not exceeding 10 ft and

(b) beams and girders where the ratio of the sum of column stiffnessess to beam stiffness exceeds 8 at each end of the span....$w(L')^2/12$

Negative moment at interior faces of exterior support for members built integrally with their supports:

Where the support is a spandrel beam or girder...................$w(L')^2/24$

Where the support is a column.............$w(L')^2/16$

SHEAR FORCES

Shear in end members at

first interior support .... $1.5w(L')/2$

Shear at all other supports .... $w(L')/2$

3.2 JOISTS AND GIRDERS

Both joists and girders are designed as simply supported beams of Tee section with open-web. Since in Ultimate Strength Design the tensile strength of concrete below the neutral axis of the cross section can be omitted except the concrete cover to the main reinforcement for tensile strength. In such a case the amount of steel and concrete required can be reduced markedly. (Fig. 3.2)

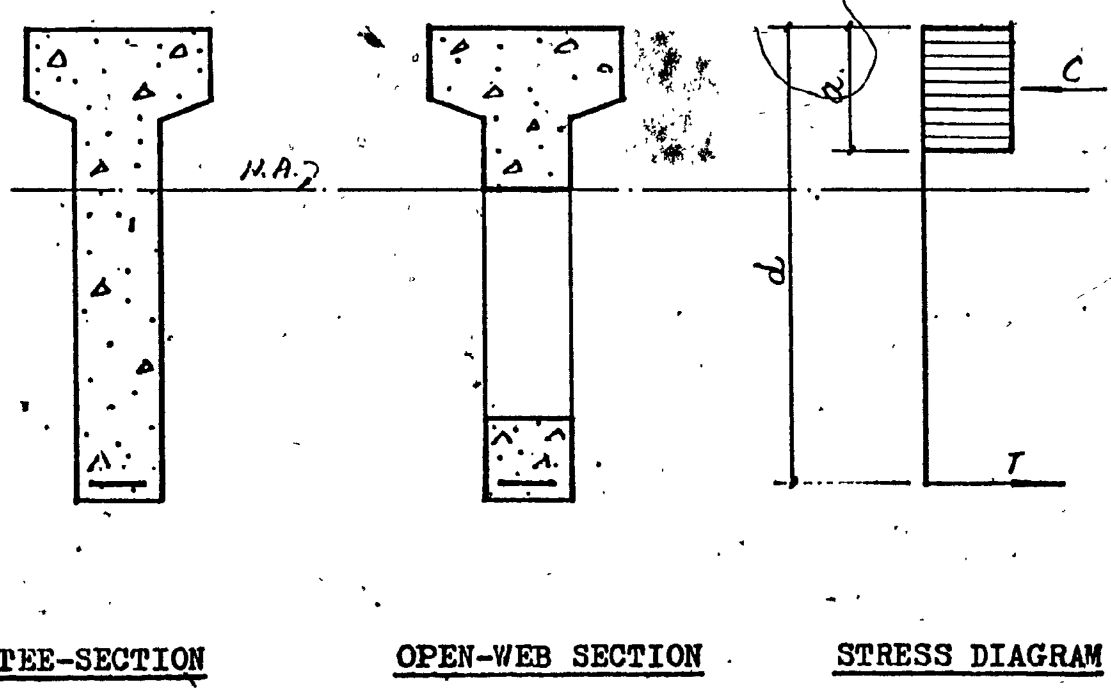


Fig. 3.2

For the Ultimate Strength Method the design of joists and girders follows the procedure and formulae as follows:

i) Determine the design load moment M_u

$$M_u = \phi_d \, M_d + \phi_l \, M_l$$

Where M_d = service dead load moment

M_l = service live load moment

ϕ_d = 1.4 for dead load

ϕ_l = 1.7 for live load

ii) Determine the depth of equivalent rectangular stress block a. (Fig. 3.2).

a = 1.18 ωd, ω can be found from

$$M_u = \phi \, bd^2 \, f'_c \, \omega \, (1 - 0.59\omega)$$

If the depth of the equivalent block is less than the depth of flange, the joist and girder may be designed as a rectangular section.

iii) Determine the area of steel necessary to develop the the equivalent stress block.

$$A_s = \frac{0.85 \, f'_c \, ab}{f_y}$$

iv) Check that reinforcement ratio does not exceed 0.75 of the ratio for balanced condition ρ_b

$$\rho_b = \frac{0.85 \, f'_c \, \beta}{f_y} \times \frac{87000}{87000 + f_y}$$

$$\text{where } \beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000}\right)$$

v) Check that reinforcement ratio is not less than the required minimum of $200/f_y$

vi) Check distribution of flexural reinforcement. ACI Code requires that when the design yield strength f_y for tension reinforcement exceeds 40,000 psi, the cross sections of maximum positive and negative moment shall be so proportioned that the quantity Z given by

$$Z = f_s \sqrt[3]{d_c A}$$

does not exceed 175 KLF for interior expoxure and 145 KLF for exterior exposure. Where d_c = distance from extreme tension fibre to the centre of the adjacent bar

A = Average effective area of concrete in tension around each reinforcing bar

f_s = Steel stress

$$f_s = 12\,(M_d + M_l)/A_s(d + kd/3)$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

The equation $Z = f_s \sqrt[3]{d_c A}$ is based on the Gergely-Lutz equation with the recommended maximum allowable crack widths, 0.016 in. for interior exposure and 0.013 in. for exterior exposure[11].

vii) Design horizontal shear. This is governed by the stresses applied and by the conditions of the interface.

The first step is to decide on the interface condition to obtain the permissible shear stress V_h. ACI Code states that when contact surfaces are clean and intentionally roughened, the allowable stress is 80 psi and when surfaces are cleaned, not intentionally roughened, but with minimum required ties used, the allowable shear stress is also 80 psi. However, when both minimum ties and intentional roughening are used, the allowable stress increases to 350 psi

When the shear stress exceeds 350 psi, the section must be checked for shear friction reinforcement as described in section 3.3.

The horizontal shear stress V_{dh} may be calculated at any cross section as

$$V_{dh} = \frac{V_u}{\phi bd}$$

When vertical bars or extended stirrups are used to transfer horizontal shear, the tie area shall not be less than that

$$A_v = 50 \frac{b_w s}{f_y}$$

and the spacing shall not exceed four times the least dimension of the supported element nor 24 in.

Besides the above concerns special care must be taken in the design of lifting devices fabricated from reinforcing bars. The inclination of the lifting force must be considered for all conditions of lifting because the 'rigging' used at the pre-casting plant is not likely to be the same as that used at the erection site, and the widths of strong backs, size of hooks, and length of lifting cables change the inclination of the lifting force.

3.3. BRACKETS

To arrange the top edges of joists and girders at the same level, brackets mounting at midspan of girders are introduced as shown in Fig. 1.1 and Fig. 3.3.

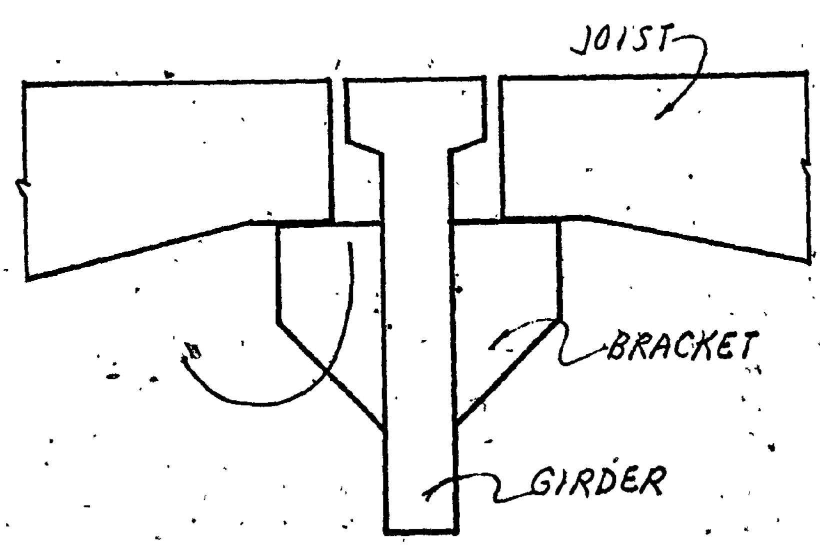


Fig. 3.3.

According to that brackets are relatively small members, details of bond, enchorage, and bearing are very important in design. The ACI Code provides good basic design rules and requirements which are based on the results of more than 200 tests. Also, modified design methods, simplified equations, and examples are given by Mattock (12) (13)

The provisions apply to brackets and corbels having a shear-span depth ratio, a/d of unity or less. In particular the ACI requires that the shear stress should not exceed:

$$V_u = \left[6.5 - 5.1\sqrt{\frac{N_u}{V_u}}\right]\left[1 - 0.5\ a/d\right]\left\{1+\left[64+160\sqrt{\left(\frac{N_u}{V_u}\right)^3}\right]\right\}\sqrt{f_c'}$$

where ρ should not exceed $0.13\ f_c'/f_y$ and N_u/V_u should not be taken less than 0.2. The tensile force N_u shall be regarded as a live load even when it has resulted from creep, shrinkage or temperature change.

When provisions are made to avoid tension due to shrinkage and creep so that the member is subject to shear and moment only, V_u should not exceed

$$V_u = \left[6.5\ (1 - 0.5\ a/d) + (1 + 64\rho_n)\right]\sqrt{f_c'}$$

where $\rho_n = \dfrac{A_s + A_{sh}}{bd}$ but not greater than $0.2\ f_c'/f_y$ and A_{sh} shall not exceed A_s .

The closed stirrups or ties parallel to the main tension reinforcement having a total cross sectional area A_{sh} not less the $0.5A_s$ shall be uniformly distributed within two-thirds of the effective depth adjacent to the main tension reinforcement.

The minimum ratio $\rho = A_s/bd$ shall not be less than $0.04\ f'_c\ /f_y$.

When the shear-span to depth ratio a/d is one-half or less the design provisions of ACI section 11.15 may be used in lieu of the equations described above. Hence the provisions apply where it is inappropriate to consider shear as a measure of diagonal tension, and particularly in design of reinforcing details for precast concrete structures.

A crack shall be assumed to occur along the shear path. Relative displacement should be considered to be resisted by friction maintained by shear friction reinforcement across the crack. This reinforcement shall be approximately perpendicular to the assumed crack (Fig. 3.4)

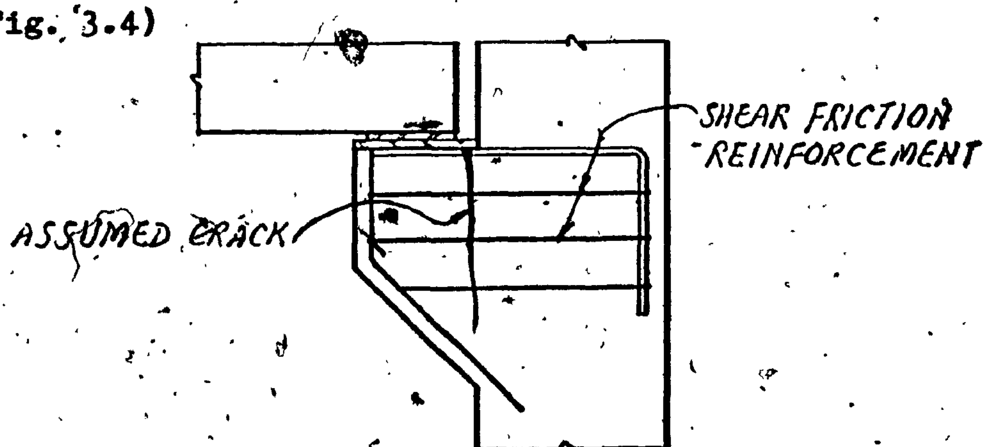


Fig. 3.4

The shear stress allowed by ACI Code shall not exceed 0.2 f'_c nor 800 psi

The required area of reinforcement A_{uf} shall be computed by

$$A_{vf} = \frac{V_u}{\phi f_y \mu} \quad \text{or} \quad A_{vf} = K_{v1} K_{v2} V_u$$

Where

$$K_{v1} = \frac{1}{\phi f_y}, \quad K_{v2} = 1.4/\mu$$

The design yield strength f_y shall not exceed 60000 psi. The coefficient of friction is listed as follow:

TABLE 11

CRACK INTERFACE CONDITION	RECOMMENDED μ	MAXIMUM V_u PSI
CONCRETE TO CONCRETE CAST MONOLITHICALLY	1.4	800
CONCRETE TO HARDENED CONCRETE, 1/4 in. ROUGHNESS	1.0	600
CONCRETE TO STEEL WITH WELDED STUDS	1.0	600
CONCRETE TO CONCRETE SMOOTH INTERFACE	0.7	420

If an axial force N_u is present, then A_{vf} should be calcualted by:

$$A_{vf} = \frac{1}{\phi f_y} \left(\frac{V_u}{\mu} + N_u \right)$$

Direct tension reinforcement across the assumed crack shall be provided. The shear-friction reinforcement shall be well distributed across the assumed crack and shall be adequately anchored on both sides by embedment, hooks, or welding to special devices.

3.4 CONNECTIONS (8) (9)

Since the design of connections in precast concrete elements is a very important part which mainly relates the strength of the whole structure, sufficient provisions should be made to ensure that all the precast elements can be connected exactly and firmly.

The reinforcement may be designed in accordance with the shear friction theory as described in Section 3.3. A basic assumption used in applying the shear friction concept is that the concrete within the connection area will crack in the most undesirable manner as shown in (Fig. 3.5). Ductility is achieved by placing reinforcement across this crack so that the tension developed by the reinforcing bars will provide a normal force to the crack. This normal force in combination with "friction" at the crack interface provides the shear resistance.

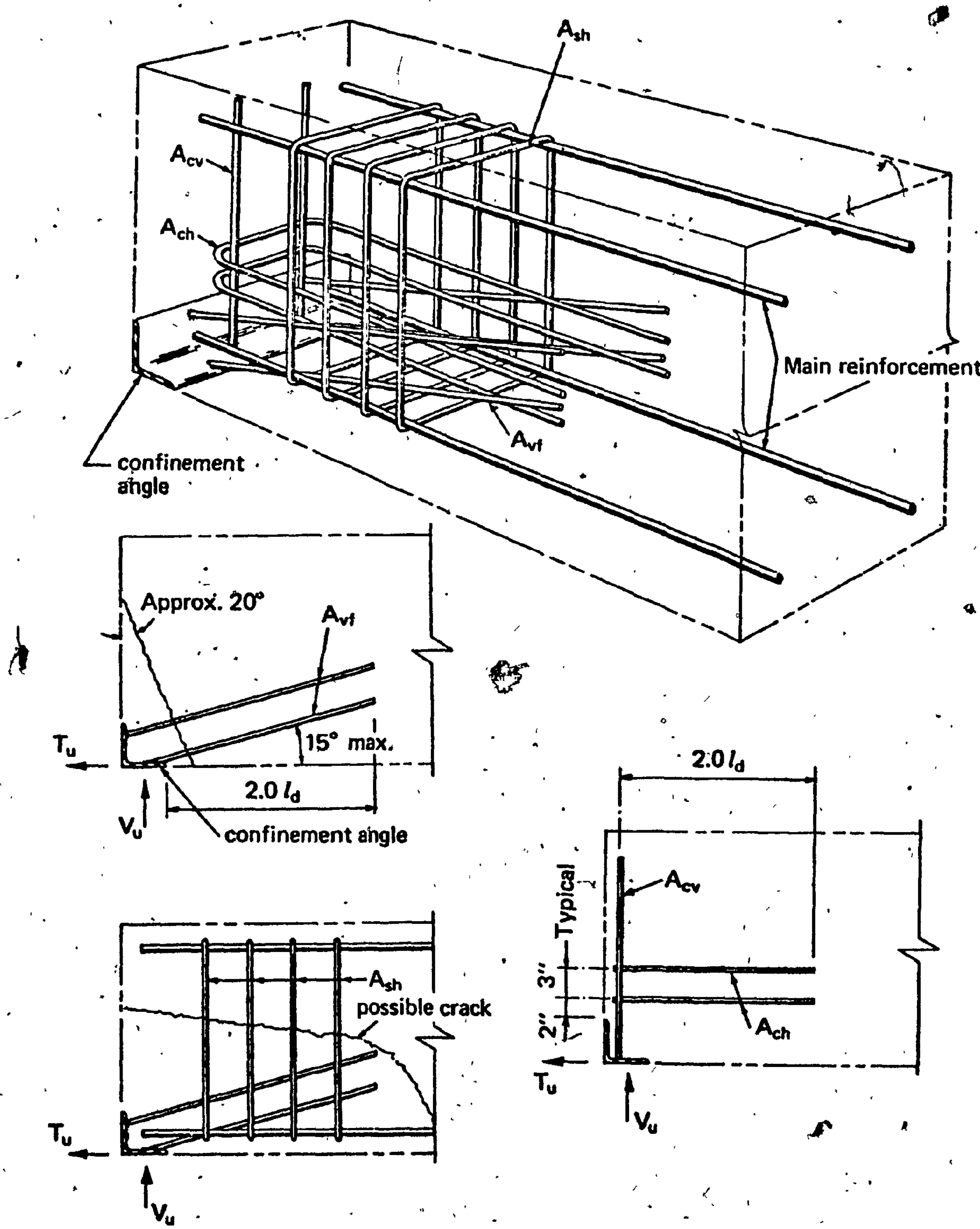


Fig. 3.5 Shear friction reinforcement (9)

The reinforcement required can be calculated with the formula as described in section 3.3

$$A_{vf} = \frac{1}{\phi f_y} \left(\frac{V_u}{\mu} + N_u \right)$$

Reinforcement across horizontal cracks can be calculated as

$$A_{sh} = \frac{A_{vf}\ f_{yv}}{f_{ys}}$$

Additional confinement reinforcement should be provided in both the vertical and horizontal directions and may be determined as

$$A_{cv} = A_{ch} = \frac{V_u}{8f_y}$$

Roof slabs particularly are exposed to wide temperature differences, which may lead to correspondingly large thermal deformations.

It has been found in observation that the thermal deformation in roof panels was four times greater than in the external walls of the top storey.

When the roof panels are rigidly attached to the structure of the top storey, such differential thermal deformations also cause cracks to appear on the external face of the building. To avoid the formation of cracks, the roof should be divided by suitably closely-spaced expansion joints.

Polish standards specify that roof structures not insulated on the upper surface must have expansion joints not further apart than 20m. The joint must be sufficiently deep to give the roof complete freedom of thermal deformations.

In buildings made of large prefabricates, particularly with shallow ventilated roofs, the fulfilment of the above requirements could be rather difficult.

The need to provide expansion joints in the upper skin of a roof structure may be obviated by laying the panels loosely on supporting girders or other beams of the main roof and leaving any joints around the panels.

The spacing of expansion joints in unventilated roof slabs should not be greater than 40 m according to the Polish standard.(14)

Experience has shown that, in prefabricated buildings with unventilated roofs, the expansion joints should be spaced even more closely (approx. 30m). Alternatively, the roof slab should be so designed that along the edges of the building, the upper skin of the roof has a degree of freedom which will allow it a measure of deformation relative to the lower insulating skin.

CHAPTER 4

APPLICATION - DESIGN EXAMPLE

A roof plan and cross sections are shown in Fig. 4.1. All panels, joists and girders are precast concrete members.

The loading and materials used are listed below.

DEAD LOAD: 50 lb/sq.ft.

LIVE LOAD: 100 lb/sq.ft. (heavy snow load condition)

CONCRETE : f'_c = 4000 psi

STEEL FOR REINFORCEMENT: f_y = 60000 psi

STEEL FOR STIRRUPS: f_g = 40000 psi

DESIGN METHOD: Ultimate Design Method.

DESIGN HANDBOOK: ACI Design Handbook Vol. 1

PCI Design Handbook

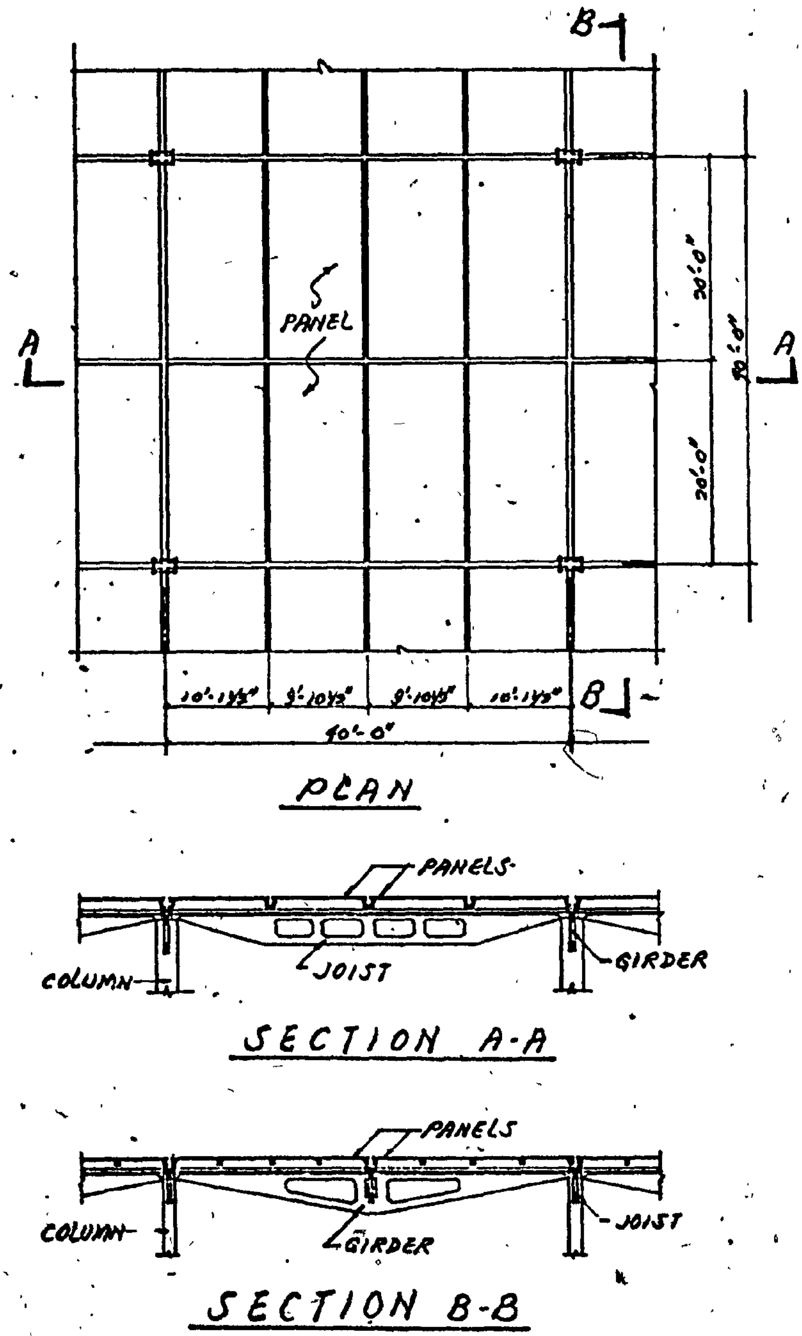


Fig. 4.1 Framing plan and sections

4.1. PANEL

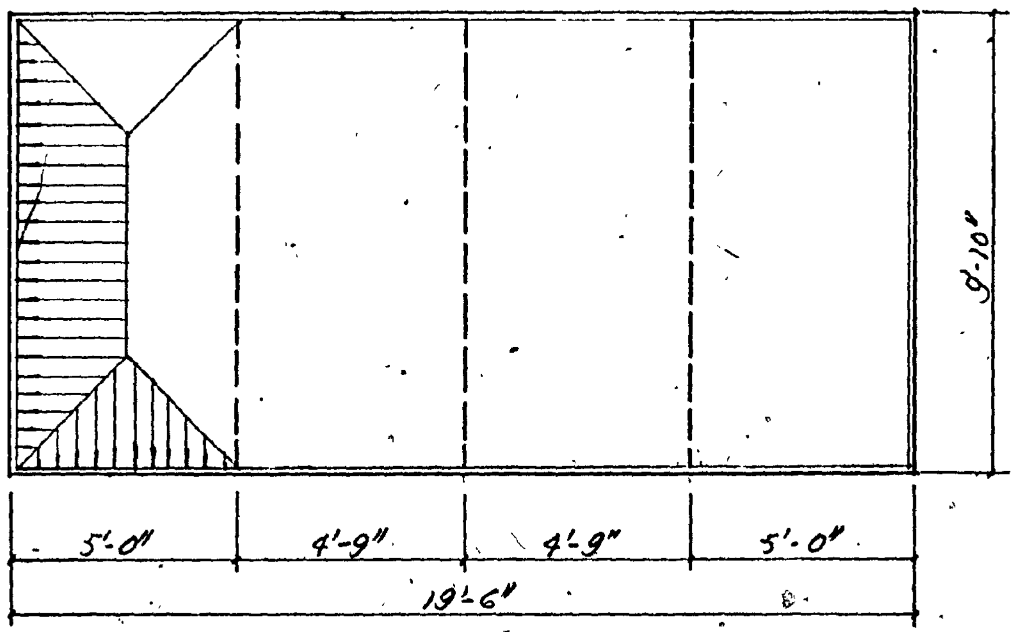


Fig. 4.2

For slab

Dead load = 50 lb/sq.ft.

Live load = 100 lb/sq.ft.

Thickness of slab h = 1.75"

Ratio m = 4.75/9 = 0.5

Uniform loading w = 1.4 x 50 + 1.7 x 100

= 2.4 lb/ft.

For short span S = 4.25' (clear span)

Using moment coeficients C from Table 1

For positive moment C = 0.074

For negative moment C = 0.098

Positive moment at mid-span

M_u = CWS = 0.074 x 240 x 4.25^2 = 320.79 ft.lb/ft.

Negative moment at support.

$$M_u = 0.098 \times 240 \times 4.25^2 = 424.83 \text{ ft.lb/ft.}$$

For balanced condition the reinforcement ratio

$$\rho_b = \frac{0.85 f_c' \beta_1}{f_y} \times \frac{87000}{87000 + f_y}$$

where $\beta_1 = 0.85$ for $f_c' = 4000$ psi

$$\beta_b = \frac{0.85 \times 4000 \times 0.85}{60000} \times \frac{87000}{87000 + 60000}$$

$$= 0.0285$$

Assume $\rho = 0.5 \rho_b = 0.5 \times 0.0285 = 0.143$

From ACI Handbook Table Flexure 1.2

For $\rho = 0.143$ we get

$K_u = 674$ $\quad a_n = 3.93$

For positive moment

$$A_s = \frac{M_u}{a_n d} = \frac{0.32}{3.93 \times 1} = 0.108 \text{ in}^2$$

For negative moment

$$A_s = \frac{0.425}{3.93 \times 0.75} = 0.108 \text{ in}^2$$

For long span $L = 8.83'$ (clear span)

positive moment at mid-span

$$M_u = 0.044 \times 240 \times 4.25^2 = 190.44 \text{ ft.lb/ft.}$$

Negative moment at supports.

$$M_u = 0.029 \times 240 \times 4.25^2 = 125.72 \text{ ft.lb/ft.}$$

For positive moment

$$A_s = \frac{0.191}{3.93 \times 1} = 0.049 \text{ in}^2$$

For negative moment

$$A_s = \frac{0.126}{3.93 \times 0.75} = 0.043 \text{ in}^2$$

use WWF 6 x 6 - 2/7 (0.108, 0.049)

Checking minimum reinforcement required by ACI code 318-71 sec. 7.13.

$$A_s = 0.0018 \times 12 \times 1.75 = 0.038 < 0.049 \quad \text{O.K.}$$

Checking shear

$$V_u = \frac{WS}{3} \times \frac{3 - M^2}{2} = \frac{240 \times 4.25}{3} \times \frac{3 - 0.5^2}{2}$$

$$= 467.5 \text{ lb/ft.}$$

$$V_s = \frac{V_u}{\phi bd} = \frac{467.5}{0.85 \times 12 \times 0.75} = 61.11 \text{ psi}$$

Allowable shear stress

$$V_s = 2\sqrt{f_c'} = 2\sqrt{4000} = 126.5 \text{ psi} > 61.11 \quad \text{O.K.}$$

4.2 RIBS

a) INTERMEDIATE RIBS

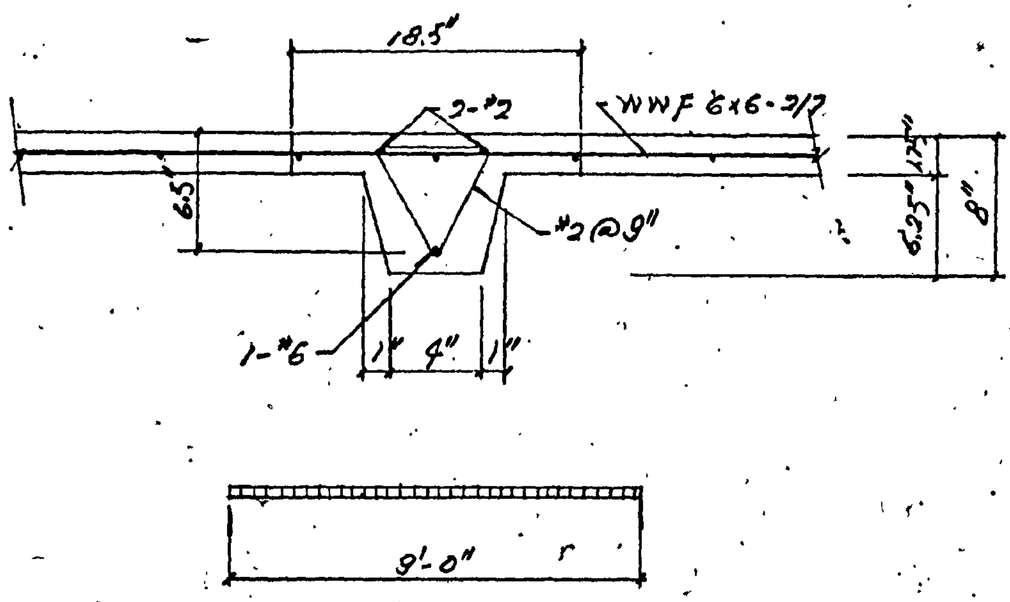


Fig. 4.3

Slab load $= \dfrac{240 \times 4.75}{3} \times \dfrac{3 - 0.5^2}{2} = 522.5$ lb/ft.

Own weight $= \frac{1}{2} \times \dfrac{4 + 6}{12} \times \dfrac{6.35}{12} \times 150 = 32.55$ lb/ft.

Total load $w = 2 \times 522.5 + 1.4 \times 32.55 = 1090.57$ lb/ft.

Resisting moment

$M_u = \frac{1}{8} \times 1090.57 \times 9^2 = 11042.05$ ft.lb. or 11.04 ft.Kip

The flange width $b = 6 + 2 \times 6.25 = 18.5"$

$b_w = 5"$ $d = 6.5"$ $h_f = 1.75"$

From ACI Handbook Table Flexure 1.2

$F = bd^2/12000 = 18.5 \times 6.5^2/12000 = 0.065$

$K_u = M_u/F = 11.04/0.065 = 169$

$\rho = 0.0032,$ $c/d = 0.066$

The ratio $h_f/d = 1.75/6.5 = 0.269 > 0.066$ (rectangular section)

The required reinforcement

$A_s = \rho bd = 0.0032 \times 18.5 \times 6.5 = 0.39 \text{ in}^2$

use 1 - #6 (0.44 > 0.39)

Checking minimum reinforcement required

$$A_s = \frac{200}{f_y} \times bd = \frac{200}{60000} \times 18.5 \times 6.5 = 0.40 \text{ in}^2 < 0.44$$

Checking shear

$V_u = 1090.57 \times 9/2 = 4907.57$ lb.

$$V_u = \frac{4907.57}{0.85 \times 5 \times 6.5} = 177.65 \text{ psi} > 126.6$$

use #2 stirrup at 9" $A_u = 0.10 \text{ in}^2$

Required shear reinforcement

$$A_u = \frac{S(V_u - V_c)b_n}{f_y} = \frac{9 \times (177.65 - 126.5) \times 5}{40000}$$

$$= 0.06 < 0.10 \quad \text{O.K.}$$

b) Transverse perimetric rib

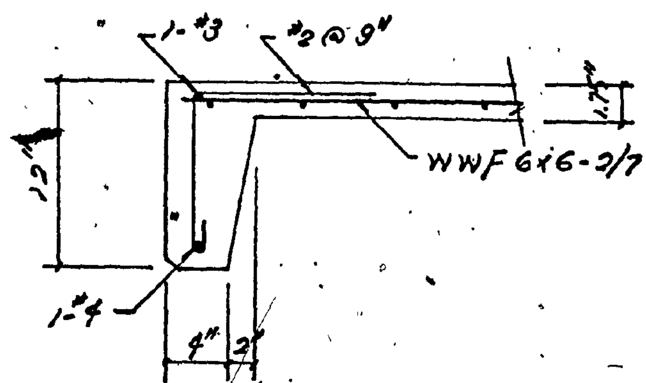


Fig. 4.4

Slab load = 522.5 lb/ft

Own weight = $\frac{1}{2} \times \frac{4 + 6}{12} \times \frac{10.25}{12} \times 150 = 53.39$ lb/ft.

Total load w = 522.5 + 1.4 x 53.39 = 597.24 lb/ft.

Resisting moment

$M_u = \frac{1}{8} \times 597.24 \times 9^2 = 6{,}047.05$ ft.lb or 6.05 ft.kip

From $b_w = 5"$ $d = 10.5"$

$F = 5 \times 10.5^2/12000 = 0.046$

$K_u = 6.05/0.046 = 131$ (ACI Table Flexure 1.2)

$\rho = 0.0025$

The required reinforcement —

$A_s = 0.0025 \times 5 \times 10.5 = 0.13\ in^2$

Use 1 - #4 (0.20 > 0.13)

Checking minimum reinforcement required

$A_s = \frac{200}{f_y} \times bd = \frac{200}{60000} \times 5 \times 10.5 = 0.18 < 0.20$

Checking shear

$V_u = 597.24 \times 9/2 = 2687.58$ lb.

$v_u = \frac{2687.58}{0.85 \times 5 \times 10.5} = 60.23 \text{ psi} < 126.5$

c) <u>Longitudinal perimetric rib</u>

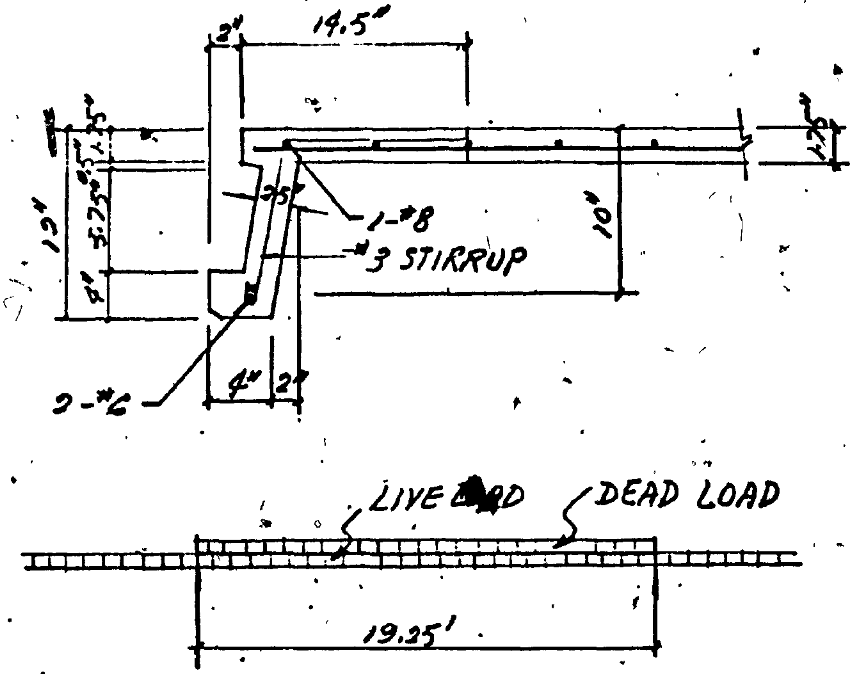


Fig. 4.5

Dead Load

$$\text{Slab } \left(\frac{50 \times 4.75}{3} \times \frac{3 - 0.5^2}{2} \times 9 \times \frac{6}{2}\right)/19.25 + \frac{50 + 4.75}{3}$$

$$= 231.85 \text{ lb/ft.}$$

$$\text{Intermediate rib } 32.55 \times 9 \times \frac{3}{2} / 19.25 = 22.83 \text{ lb/ft.}$$

$$\text{Own weight } \left(\frac{2.5}{12} \times \frac{6.25}{12} + \frac{4}{12} \times \frac{4}{12}\right) \times 150 = 32.94 \text{ lb/ft.}$$

$$\text{Total} = 287.62 \text{ lb/ft.}$$

$$M_u = 1.4 \times \frac{1}{8} \times 287.62 \times 19.25^2 = 18651.71 \text{ ft-lb}$$

$$\text{or} \quad 18.65 \text{ ft-kip}$$

Live load

$$\left(\frac{100 \times 4.75}{3} \times \frac{3 \text{ [illegible] } 0.5^2}{2} \times 9 \times 6/2\right)/19.25 + \frac{100 \times 4.75}{3}$$

$$= 463.69 \text{ lb/ft.}$$

Since the longitudinal perimetric rib is designed as a continuous beam for live load by extending the main bars beyond both ends of the rib, the positive and negative moment coefficients are obtained as described in section 3.1.

Positive moment at midspan

$$M_u = 1.7 \times \frac{1}{16} \times 463.69 \times 19.25^2 = 18256.53 \text{ ft-lb.}$$

$$\text{or} \quad 18.26 \text{ ft-kip}$$

Negative moment at support

$$M_u = 1.7 \times \frac{1}{11} \times 463.69 \times 19.25^2 = 26550.00 \text{ ft-lb.}$$

$$\text{or} \quad 26.55 \text{ ft-kip}$$

Total positive moment

$$M_u = 18.65 + 18.26 = 36.91 \text{ ft-kip}$$

$b = 14.5"$ $\quad b_w = 2.5"$ $\quad d = 10"$ $\quad h_f = 1.75"$

$F = 14.5 \times 10^2/12000 = 0.121$ (ACI Table Flexure 1.2)

$K_u = 36.91/0.121 = 305$

$\rho = 0.0060 \qquad c/d = 0.125$

$h_f/d = 1.75/10 = 0.175 > 0.125$ (rectangular section)

$A_s = 0.006 \times 14.5 \times 10 = 0.87\ in^2$

use 2 - #6 $(0.88 > 0.87)$

For negative moment

$b = 4" \qquad d = 10.5" \qquad h_f = 4"$

$F = 4 \times 10.5^2/12000 = 0.037$

$K_u = 26.55/0.037 = 722$

$\rho = 0.0155 \qquad c/d = 0.323$

$h_f/d = 4/10.5 = 0.381 > 0.323$ (rectangular section)

$A_s = 0.0155 \times 4 \times 10.5 = 0.65\ in^2$

use 1 - #8 $(0.79 > 0.65)$

Checking shear

$$V_u = (1.4 \times 287.62 + 1.7 \times 463.69) \times \frac{19.25}{2}$$

$$= 11462.81 \text{ lb}$$

$$v_n = \frac{11462.81}{0.85 \times 2.5 \times 10.5} = 513.74 \text{ psi} > 126.5$$

Shear reinforcement is required

Use #2 stirrups. $\qquad A_v = 0.11$

1'-0" from support

$$V_u = 1146.81 - (1.4 \times 287.62 + 1.7 \times 463.69)$$

$$= 10{,}271.87 \text{ lb.}$$

$$V_u - V_c = 10271.87 - 126.5 \times 2.5 \times 10.5$$

$$= 7{,}449.34 \text{ lb.}$$

The spacing of stirrups

$$S = \frac{\phi A_u f_y d}{V_u - V_c} = \frac{0.85 \times 0.11 \times 40000 \times 10.5}{7449.34}$$

$$= 5.27'' \quad \text{use } 5''$$

2'-0" from support

$$V_u = 11462.81 - (1.4 \times 287.62 + 1.7 \times 463.69) \times 2$$

$$= 9080.93 \text{ lb.}$$

$$V_u - V_c = 9080.93 - 0.85 \times 126.5 \times 2.5 \times 10.5$$

$$= 6258.40 \text{ lb.}$$

$$S = \frac{0.85 \times 0.11 \times 40000 \times 10.5}{6258.40} = 6.27'' \quad \text{use } 6''$$

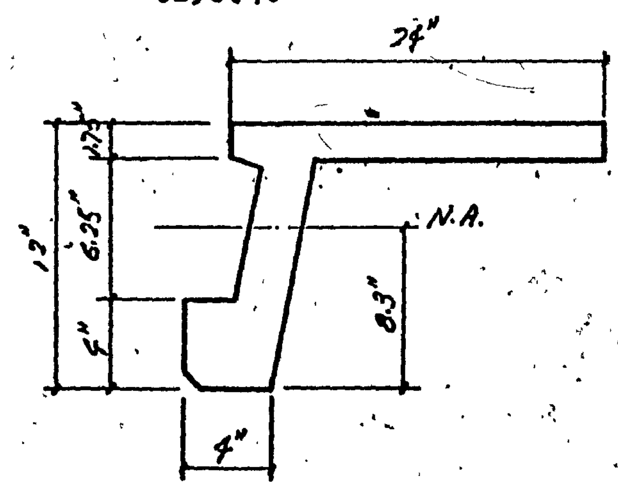


Fig. 4.6

a) Section at mid-span

$b_e = 24"$

$$n = E_s/E_c = \frac{29 \times 10^6}{57000 \sqrt{4000}} = 8$$

To find the neutral axis location

$$x = \frac{2.5 \times 6.25 \times 7.125 + 24 \times 1.75 \times 11.125 \times 4 \times 4 \times 2}{2.5 \times 6.25 + 24 \times 1.75 + 4 \times 4}$$

$= 8.3"$

The moment of inertia of gross section

$$I_g = 1/3 \times 2.5 \,(4.3^3 + 1.95^3) + 1/12 \times 24 \times 1.75^3 + 24 \times 1.75 \times 2.825^2 + 1/12 \times 4 \times 4^3 + 4 \times 4 \times 6.3^2$$

$= 1074.71 \text{ in}^4$

The deflection due to dead load

$$\Delta_d = \frac{5\,wl^4}{384\,EI} = \frac{5 \times 287.62 \times 19.25^4 \times 12^3}{384 \times 57000 \sqrt{4000} \times 1074.71}$$

$= 0.23$ in

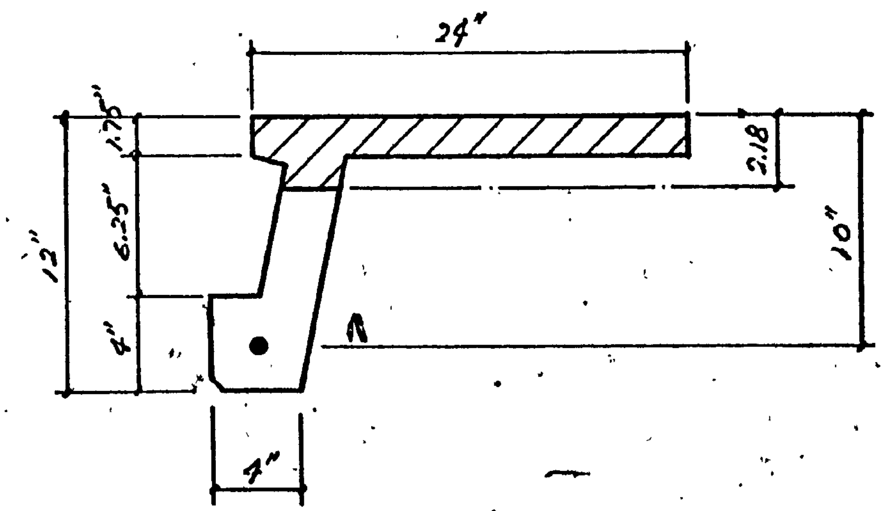


Fig. 4.7

To obtain the transformed section moment of inertia:

Find the neutral axis location,

$$24 \times 1.75\ (x - 0.875) + 2.5\ (x-1.75)^2/2 = 0.88 \times 8(10-x)$$

$$1.25\ x^2 + 44.665\ x - 103.322 = 0$$

$$x = 2.18 \text{ in}$$

$$I_{cr} = 1/12 \times 24 \times 1.75^3 + 24 \times 1.25 \times (2.18-0.875)^2$$
$$+ 1/3 \times 2.5\ (2.18-1.75)^3 + 0.88 \times 8\ (10-2.18)^2$$
$$= 512.83 \text{ in}^4$$

Cracking moment

$$M_{cr} = \frac{f_s\ I_g}{y_t}$$

Where $f_s = 7.5 \sqrt{f'_c} = 7.5 \sqrt{4000} = 474.34$ psi

Hence $M_{cr} = \dfrac{474.34 \times 1074.71}{8.3 \times 12000} = 5.12$ ft-kip

The maximum moment at midspan

$$M_{max} = 1/8\ (287.62 + 1/16 \times 463.69) \times 19.25^2$$
$$= 24061.78 \text{ ft. lb. or } 24.06 \text{ ft-kip}$$

$$\frac{M_{cr}}{M_{max}} = \frac{5.12}{24.06} = 0.22 \qquad \left(\frac{M_{cr}}{M_{max}}\right)^3 = 0.01$$

The effective moment of inertia

$$I_e = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr}$$

$$= 0.01 \times 1074.71 + (1-0.01) \times 512.83 = 518.5 \text{ in}^4$$

b) Section at supports.

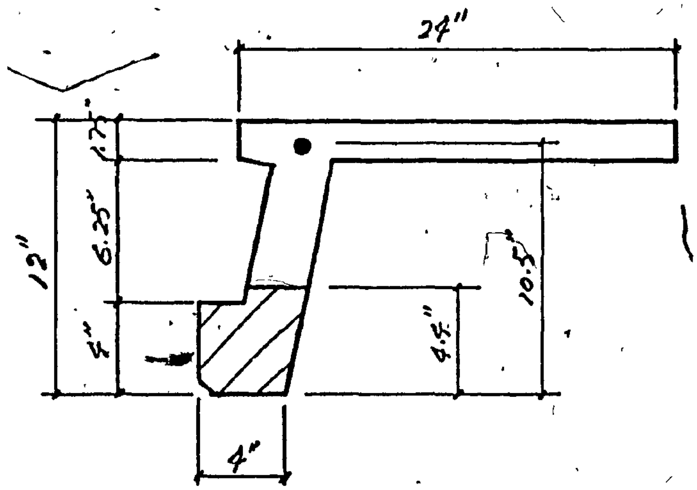


Fig. 4.8

To find the neutural axis location

$$4 \times 4\ (x - 2) + 2.5\ (x-4)^2/2 = 0.78 \times 8\ (10.5 - x)$$

$$1.25\ x^2 + 12.32\ x - 78.36 = 0$$

$$x = 4.4 \text{ in}$$

$$I_{cr} = 1/12 \times 4 \times 4^3 + 4 \times 4 \times (4.4 - 2)^2 + 0.79 \times 8\ (10.5 - 4.4)^2 + 1/3 \times 2.5\ (4.4 - 4)^3 = 350.60 \text{ in}^4$$

Cracking moment

$$M_{cr} = \frac{474.34 \times 1074.71}{3.7 \times 12000} = 11.48 \text{ ft - kip}$$

$$M_{max} = 1/11 \times 463.69 \times 19.25^2 = 15620.56 \text{ ft-lb}$$

or 15.62 ft - kip

$$\left(\frac{M_{cr}}{M_{max}}\right) = \frac{11.48}{15.62} = 0.74 \qquad \left(\frac{M_{cr}}{M_{max}}\right)^3 = 0.405$$

The effective moment of inertia

$$I_e = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr}$$

$$= 0.405 \times 1074.71 + (1 - 0.405)350.60 = 643.88 \text{ in}^4$$

The average effective moment of inertia

$$I_a = \tfrac{1}{2}(518.5 + 643.88) = 581.19 \text{ in}^4$$

The immediate deflection due to dead load and live load

$$\Delta_{(d+l)} = \frac{5L^2}{48EI}\left[M_s - \frac{1}{10}(M_a + M_b)\right]$$

$$= \frac{5 \times 19.25^2 \times 12^2}{48 \times 57000\sqrt{4000} \times 581.19}\left[24.06 - \frac{2}{10} \times 15.62\right]$$

$$\times 12000$$

$$= 0.67 \text{ in}$$

Final deflection

$$\Delta_1 = 0.67 - 0.23 = 0.44 \text{ in}$$

The allowable deflection

$$\Delta = \frac{L}{480} = \frac{19.25 \times 12}{480} = 0.48 > 0.44 \qquad \text{O.K.}$$

Checking distribution of flexural reinforcement for

$f_y = 60{,}000 \text{ psi} > 40{,}000 \text{ psi}$

The distance from extreme fibre of concrete to the centre of the adjacent bar

$$d_c = 2''$$

The average effective tension area of concrete

$$A = (2 + 1 + 2) \times 2.5/2 = 6.25 \text{ in}^2$$

$$f_s = (M_d + M_l)12/(d - kd/3)A_s$$

$$\text{where} \quad k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

$$= \sqrt{\left(8 \times \frac{0.88}{2.5 \times 10}\right)^2 + \left(2 \times 8 \times \frac{0.88}{2.5 \times 10}\right)}$$

$$- 8 \times \frac{0.88}{2.5 \times 10} = 0.52$$

$$\therefore \quad f_s = 24.06 \times 12/0.88\,(10 - 0.52 \times 10/3)$$

$$= 39.69$$

$$z = f_s \sqrt[3]{d_c A} = 39.69 \sqrt[3]{2 \times 6.25}$$

$$= 92.11 < 175 \qquad \text{O.K.}$$

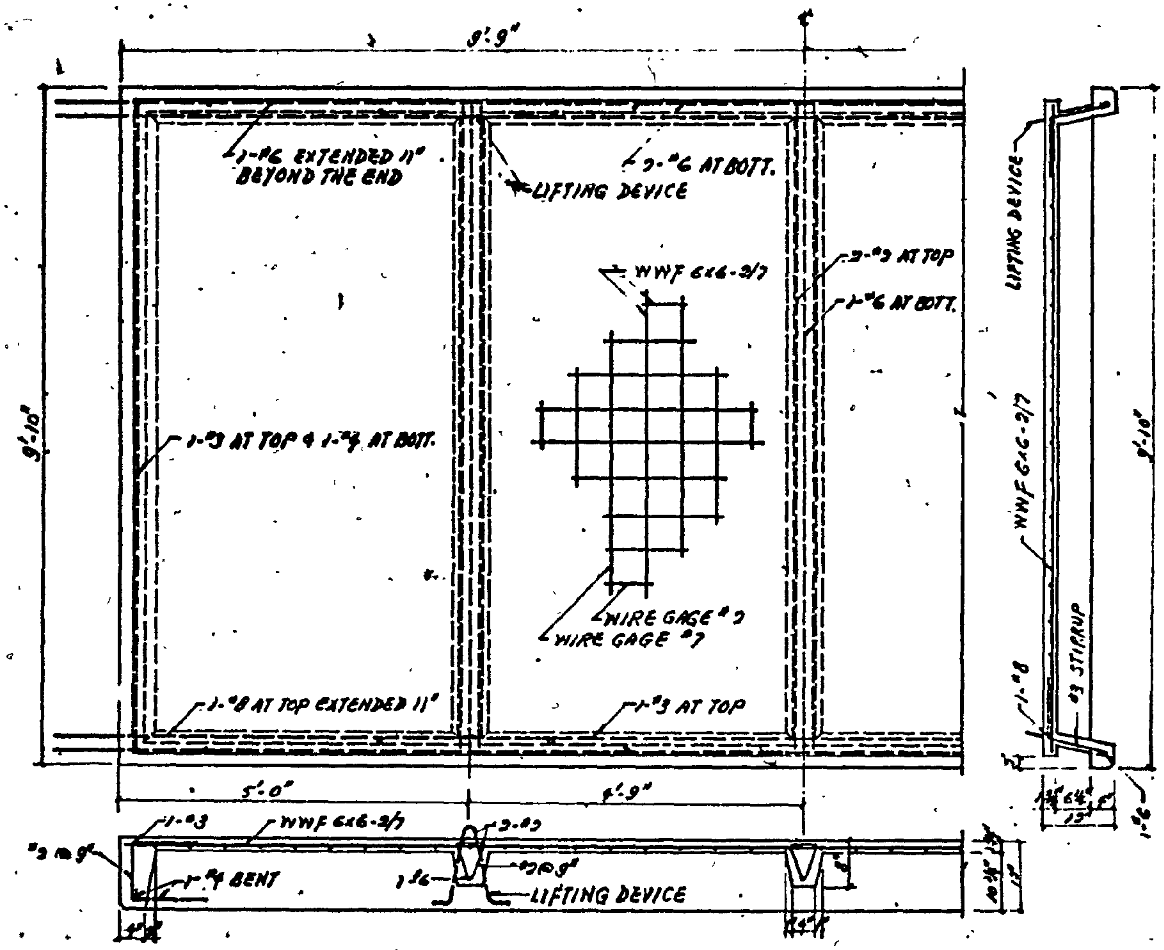


Fig. 4.9 Plan and sections of panel

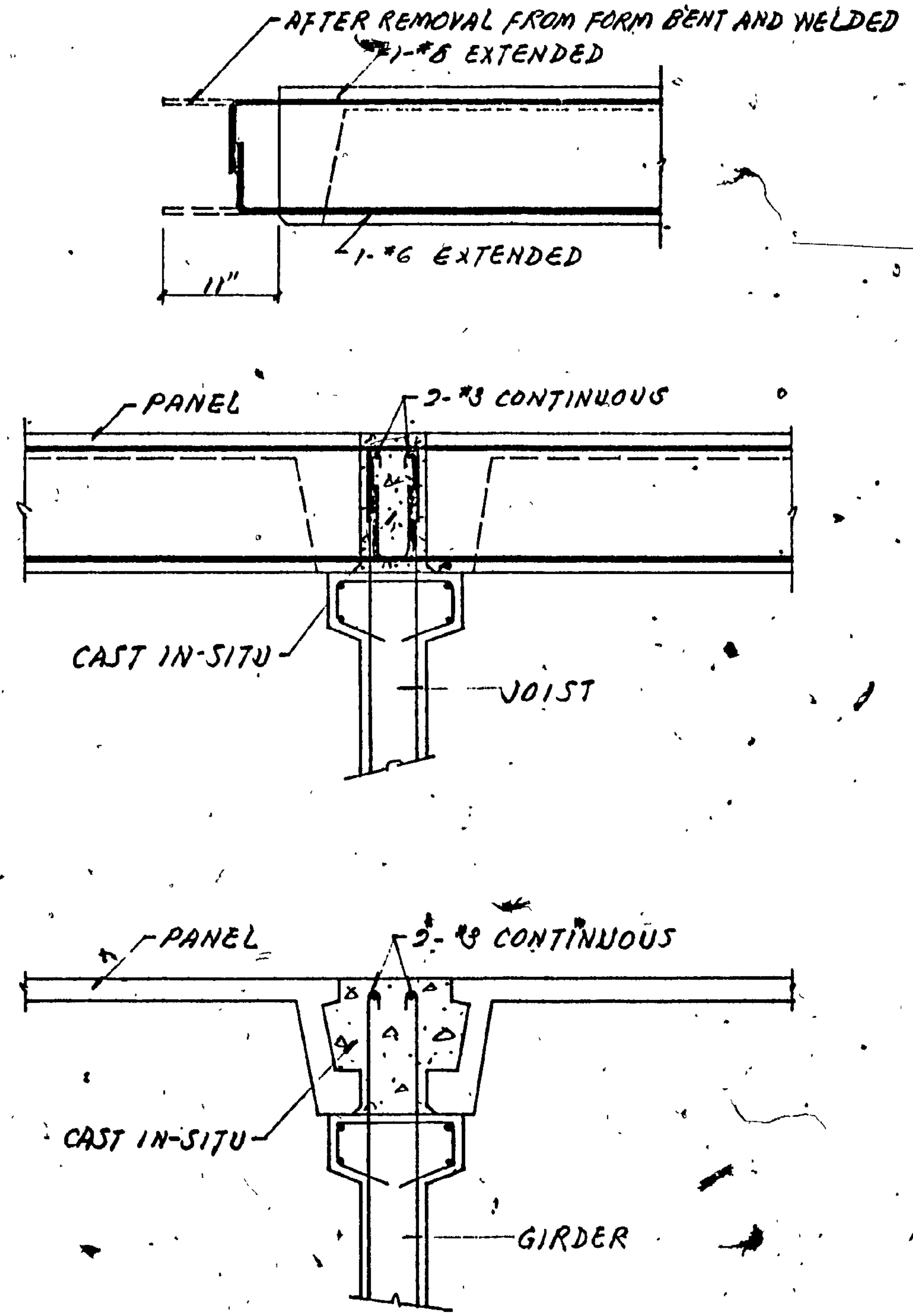


Fig. 4.10 Connection between panels, joists and girders

4.3 JOIST

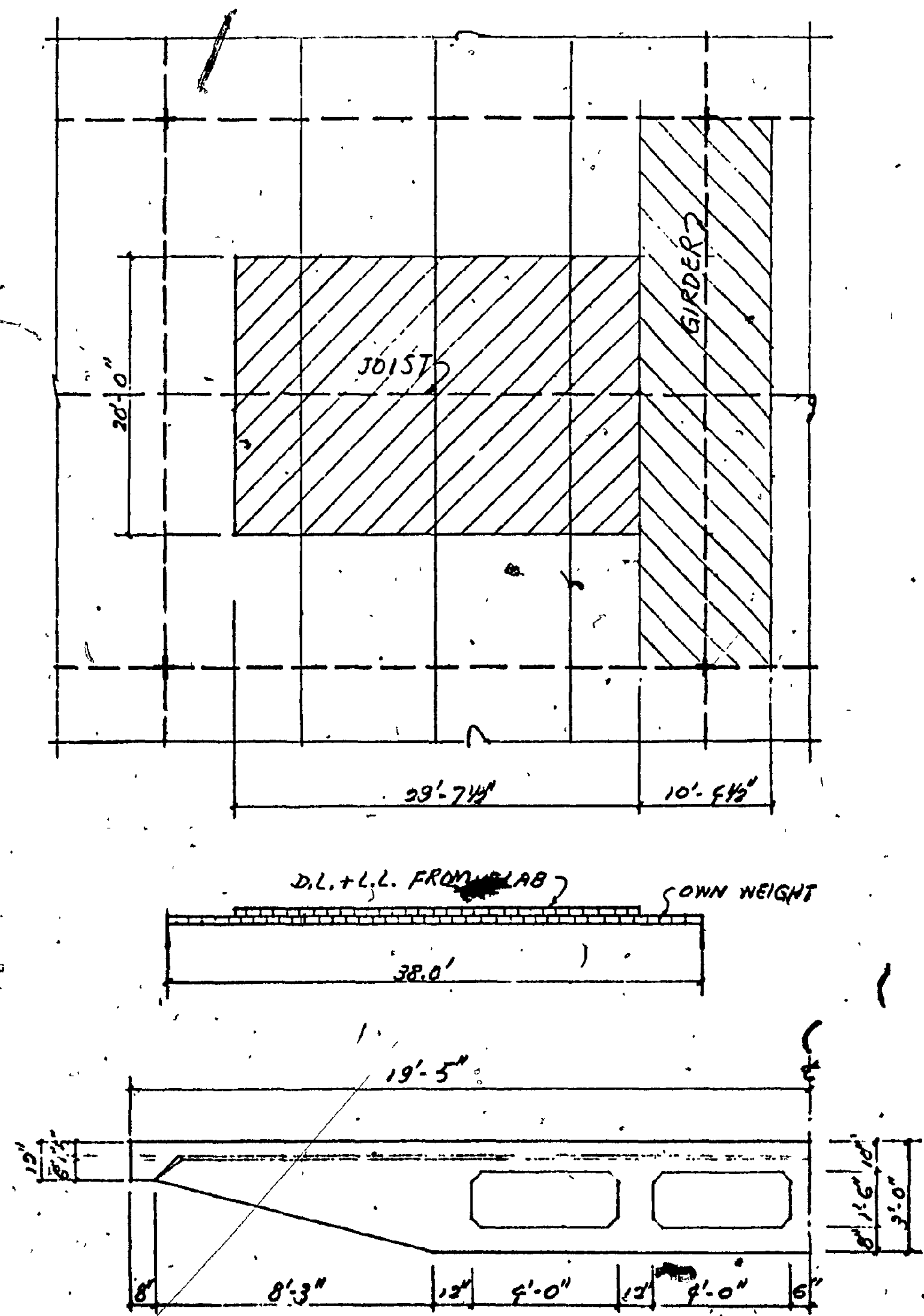


Fig. 4.11 Joist

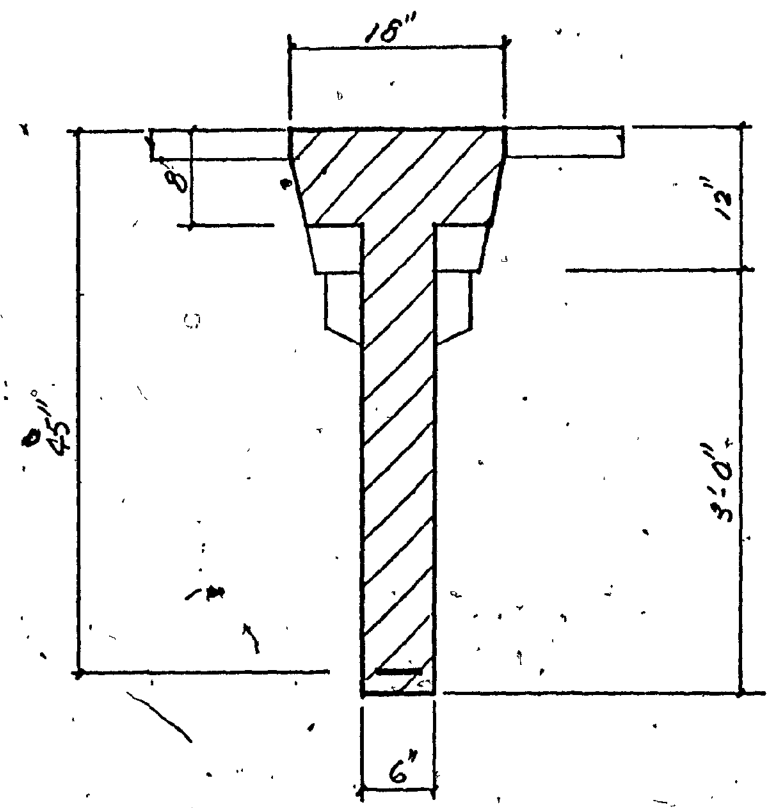


Fig. 4.12

Dead load:

$$20 \times 50 + \frac{6(32.94 \times 19.5 + 53.39 \times 9) + 9(32.55 \times 9)}{29.63}$$

$$= 1316.35 \text{ lb/ft}$$

Own weight:

$$\left[38.83 \left(5 \times 0.5 + \frac{5+6}{12} \times \frac{3}{12}\right) - 4 \times 4 \times 1.5 \times 0.5 \right.$$

$$\left. - 16.17 \times 2 \times 0.5 \right] \times 150/38.83$$

$$= 225.55 \text{ lb/ft}$$

Live load:

$$20 \times 100 = 2000 \text{ lb/ft}$$

Resisting moment:

$$M_u = 1.4 \left[\frac{1}{8} \times 225.55 \times 38^2 + \frac{1}{8} \times 1316.35 \times 29.63\ (2 \times 38 - 29.63)\right] + 1.7 \times \frac{1}{8} \times 2000 \times 29.63\ (2 \times 38 - 29.63)$$

$$= 957425.55 \text{ ft-lb}$$

$$\text{OR} = 957.43 \text{ ft-kip}$$

$b = 17"$ $b_w = 6"$ $d = 45"$ $h_f = 8"$

$F = 17 \times 45^2/12000 = 2.87$

$K_u = 957.43/2.87 = 334$

$\rho = 0.0066$ $c/d = 0.136$ (ACI Table Flexure 1.2)

$h_f/d = 8/45 = 0.178 > 0.136$ (Rectangular Section)

$\therefore$ $A_s = 0.0066 \times 17 \times 45 = 5.05 \text{ in}^2$

Use 4 - #10 $(5.08 > 5.05)$

Checking horizontal shear

The design shear load.

$$V_u = \left[1.4\ (1316.35 \times 29.63 + 225.55 \times 38) + 1.7 \times 2000 \times 38\right] / 2$$

$$= 97{,}902.05 \text{ lb.}$$

The horizontal shear stress

$$v_{dh} = \frac{V_u}{\emptyset bd} = \frac{97902.05}{0.85 \times 6 \times 45} = 426.59 > 350 \text{ psi}$$

The maximum allowable shear stress

$$v_u = 0.2\ f_c' = 0.2 \times 4000 = 800 \text{ psi} > 426.5$$

Hence shear friction reinforcement is required. The 'reduced capacity' horizontal shear between support and center line of the span is

$$\frac{V_u}{\emptyset} = \tfrac{1}{2} \times 426.59 \times 19 \times 12 \times 6 = 291{,}787.56 \text{ lb.}$$

The total shear

$$V_u = 0.85 \times 291.787.56 = 248{,}019.43 \text{ lb.}$$

Required area of reinforcement

$$A_{vf} = \frac{V_u}{\emptyset\ f_y\ \mu}$$

For concrete placed against hardened concrete

$\mu = 1.0$ (Table II)

$f_y = 40000$ psi

$\phi = 0.85$

$$\therefore A_{vf} = \frac{2480/9143}{0.85 \times 40000 \times 1.0} = 7.29 \text{ in}^2$$

Use #3 extended stirrup $A_v = 0.22 \text{ in}^2$

The required number of stirrups

$$N = \frac{A_{vf}}{A_v} = \frac{7.29}{0.22} = 33.16 \qquad \text{Use } 34$$

The spacings of these stirrups will be arranged according to the calculations in horizontal shear, vertical shear and connection. (Fig. 4. 13)

Checking vertical shear

$V_u = 97902.05$ lb.

1" - 0" from support

$V_u = 97902.05 - 1.4 \times 225.55 \times \frac{14}{12} = 97533.65$ lb

Nominal shear stress in concrete

$$V_u = \phi \; 2 \sqrt{f'_c} = 0.85 \times 2 \times \sqrt{4000} = 107.5 \text{ psi}$$

Shear force carried by concrete

$V_u' = 107.5 \times 6 \times 14 = 9030.00$ lb

Shear force carried by inclined main bars

$V_u'' = f_y\ As \sin\alpha = 60{,}000 \times 5.08 \times 0.24 = 73{,}152.00$ lb

Use #3 stirrups $A_v = 0.22\ in^2$ $f_y = 40000$ psi

The spacing of stirrups

$$S = \frac{\phi A_v f_y d}{(V_u - V_u' - V_u'')} = \frac{0.85 \times 0.22 \times 40000 \times 14}{97533.65 - 9030 - 73152} = 6.83''$$

Use 6" 9'-6" from support

$$V_u = 97902.05 - 1.4\,(225.55 \times 9.5 + 1316.35 \times 5.32) - 1.7 \times 2000 \times 5.32$$
$$= 67{,}010.06 \text{ lb.}$$

$V_u' = 107.5 \times 6 \times 45 = 29025.00$ lb

Use # 3 stirrup

The required spacing

$$S = \frac{0.85 \times 0.22 \times 40000 \times 45}{67010.06 - 29025} = 8.86'' \text{ use } 8.5''$$

Checking distribution of flexural reinforcement

$d_c = 2''$

$A = (2 + 2.25 + 2) = 6.25''$

$$f_s = \frac{13\ (M_d + M_l\ )}{A_s\ (\ d - kd/3)}$$

$M_d = 266.79 \qquad M_l = 343.49$

$$k = \sqrt{(n\rho\ )^2 + 2n\rho} - n\rho \ = \sqrt{(8\ x \frac{5.08}{6x45}\ )^2 - 2x8x\ \frac{5.08}{6x45}}$$

$$- 8\ x\ \frac{5.08}{6x45} = 0.42$$

$$f_s = \frac{12\ (266.79 + 343.49)}{5.08\ (45-0.42\ x\ 45/3)} = 37.25\ psi$$

$$Z = f_s\ \sqrt[3]{d_cA} = 37.25\ \sqrt[3]{2x6.25} = 56.75 < 175$$

O.K.

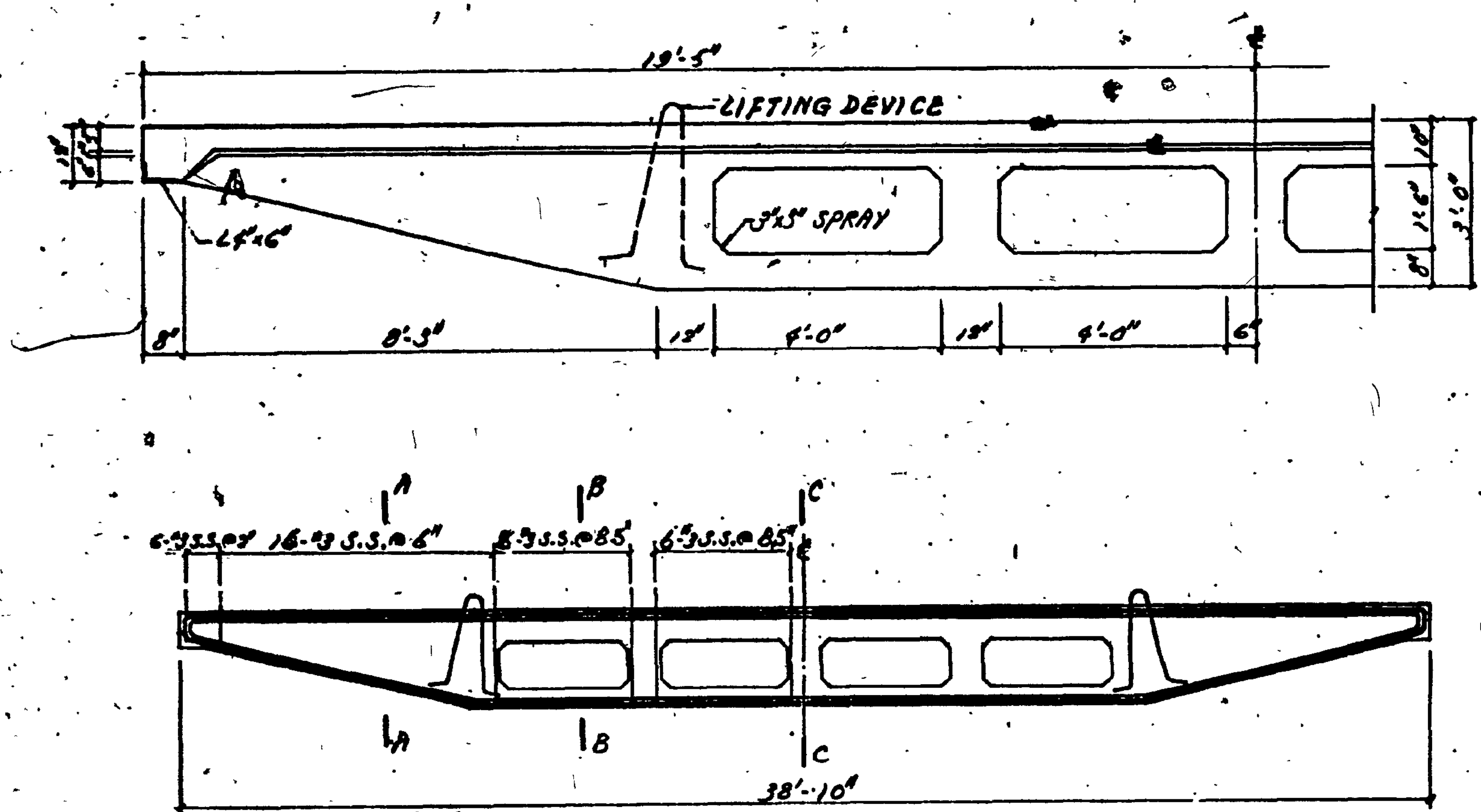


Fig. 4.13 Elevations of joist

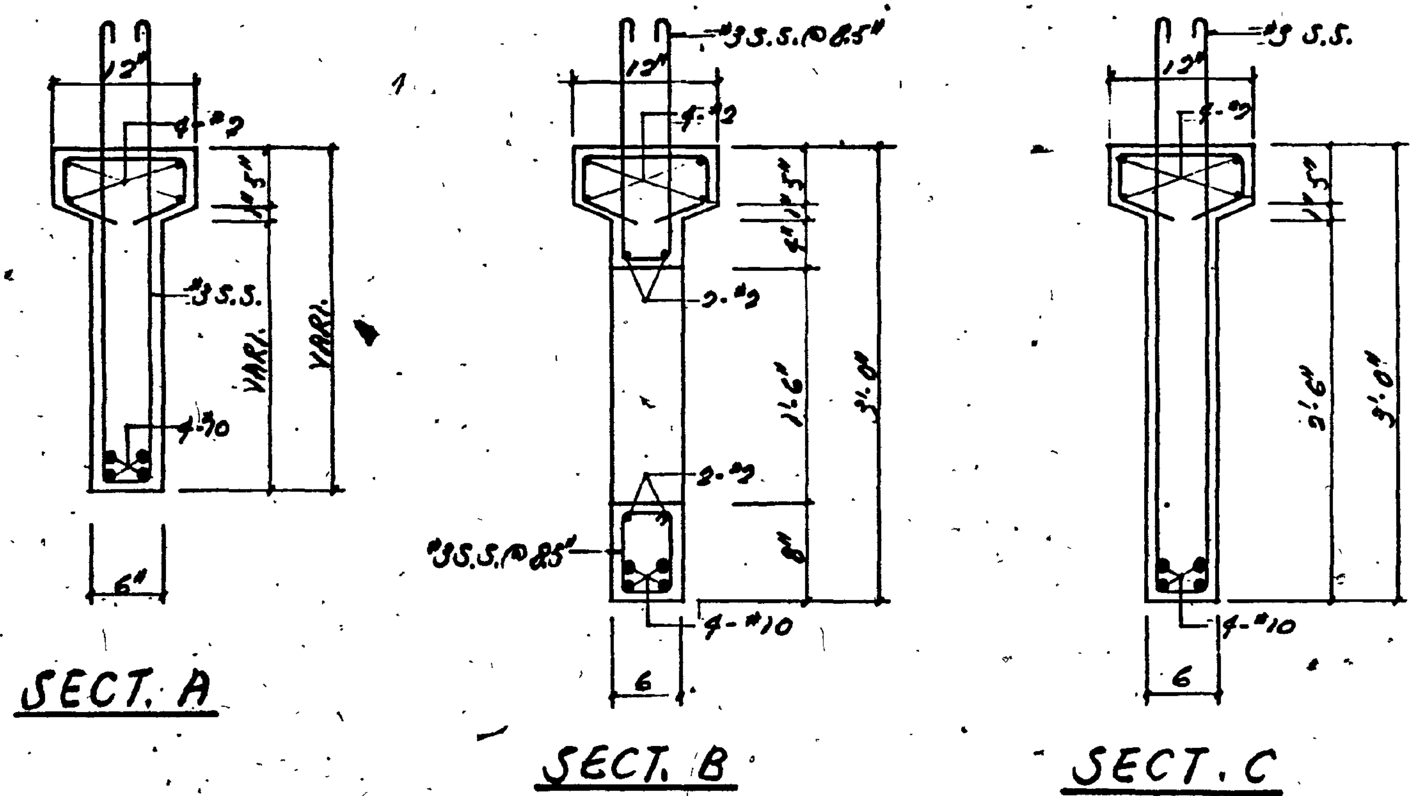
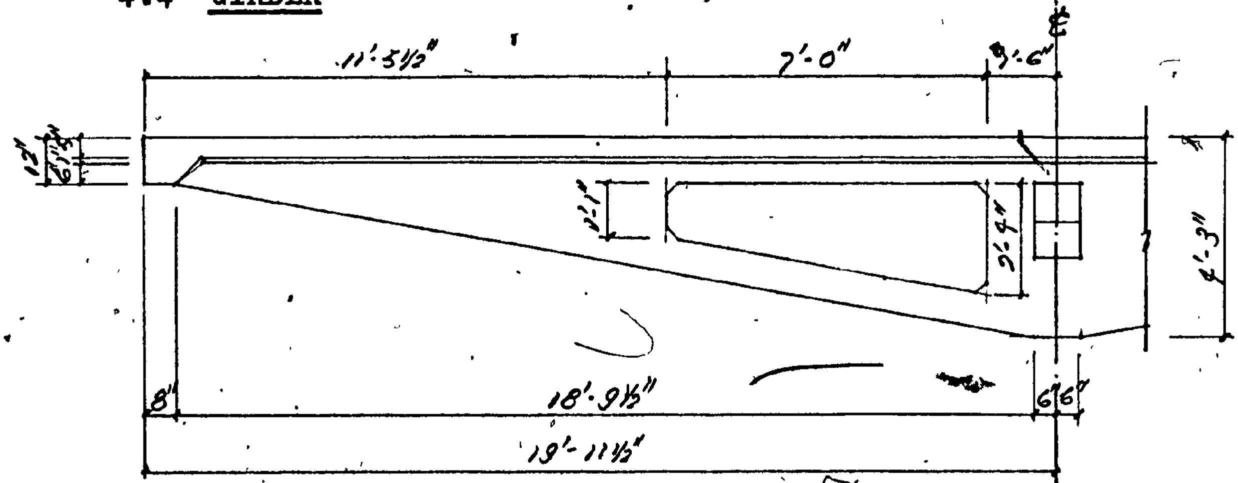


Fig. 4.14 Details of reinforcement of joist

4.4 GIRDER



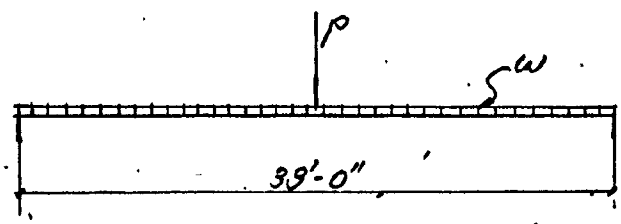


Fig. 4.15

Dead load:

Concentrated load from joist and bracket (assumed 500 lb.)

$$P_1 = 1316.35 \times 29.63 + 225.55 \times 38.83 + 500$$
$$= 48261.56 \text{ lb.}$$

$$w_1 = \frac{50 \times 40 \times 10.38 + 4(53.39 \times 9 + 32.94 \times 19.5)}{39.92}$$
$$+ \frac{6(32.55 \times 9)}{39.92}$$
$$= 676.58 \text{ lb/ft}$$

Own weight of the girder

$$w_2 = \left[5.25 \times 0.5 + \frac{5+6}{12} \times \frac{3}{12} - \frac{(1.08 + 2.33) \times 7 \times 0.5 \times 18.79 \times 3.25 \times 0.5}{39.92}\right] \times 150$$

$$= 268.55 \text{ lb/ft}$$

Live load:

concentrated load from joist

$$P_2 = 20 \times 29.63 \times 100 = 59{,}260 \text{ lb}$$

$$w_3 = 100 \times 10.38 = 1{,}038 \text{ lb/ft}$$

Resisting moment:

$$M_u = \tfrac{1}{4}(1.4 \times 48{,}261.56 + 1.7 \times 59{,}260) \times 39$$

$$+ 1/8 \left[1.4(676.58 + 268.55) + 1.7 \times 1{,}038\right] \times 39^2$$

$$= 2{,}228{,}069.35 \text{ ft-lb}$$

or 2,228.07 ft-kip

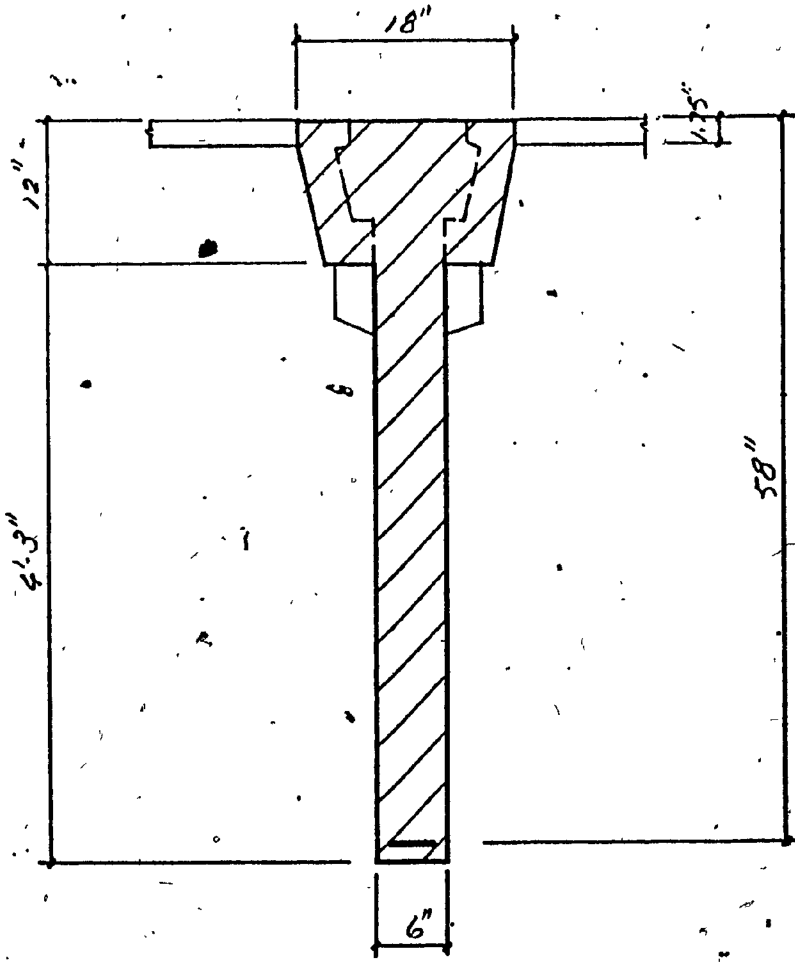


Fig. 4.16

$b = 17"$ $d = 58"$ $b_w = 6"$ $h_f = 12"$

$F = 17x58^2 / 12000 = 4.77$

$k_u = 2,228.07/4.77 = 468$

$\rho = 0.0094$ $\qquad$ $c/d = 0.197$

$h_{f/d} = 12/58 = 0.207 < 0.197$ (Rectangular section)

$A_s = 0.0094 \times 17 \times 58 = 9.29\ in^2$

Use 6 - #11 (9.36 > 9.27)

Checking horizontal shear:

The design shear load

$$V_u = \left\{ 1.4 \left[48,261.56 + (676.58 + 268.55) \times 39 \right] + 1.7\,(59,260 + 1,038 \times 39) \right\} /2 = 144,365.84\ \text{lb}$$

The horizontal shear stress

$$V_{dh} = \frac{V_u}{\phi\, bd} = \frac{144,365.84}{0.85 \times 6 \times 58} = 488.05\ \text{psi} > 350\ \text{psi}$$

The maximum allowable shear stress

$$V_u = 0.2\, f'_c = 0.2 \times 4000 = 800 > 488.05\ \text{psi}$$

Hence shear friction reinforcement is required. The 'reduced capacity' horizontal shear between support and centerline of this span is

$$\frac{V_u}{\phi} = 1/2 \times 488.05 \times 19.5 \times 12 \times 6 = 342,611.10\ \text{lb.}$$

$$V_u = 0.85 \times 342,611.10 = 291,219.44\ \text{lb.}$$

Required area of reinforcement

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{291219.44}{0.85 \times 40000 \times 1.0} = 8.58 \text{ in}^2$$

Use #3 stirrups $A_v = 0.22$

The required number of stirrups

$$N = \frac{A_{vf}}{A_v} = \frac{8.58}{0.22} = 39$$

Checking vertical shear

$$V_u = 144{,}365.84 \text{ lb}$$

1' - 0" from support

$$V_u = 144{,}365.84 - \left[1.4(676.58 + 268.55) + 1.7 \times 1038\right]$$
$$= 138{,}962.22 \text{ lb.}$$

Shear force carried by concrete

$$V_u' = 107.5 \times 6 \times 13 = 8385.00 \text{ lb.}$$

Shear force carried by inclined main bars

$$V_u'' = 9.36 \times 60{,}000 \times 0.17 = 95{,}472.00 \text{ lb.}$$

Use # 3 stirrup $A_v = 0.22$

The spacing of stirrups

$$S = \frac{0.85 \times 0.22 \times 40000 \times 13}{138962.22 - 8385 - 95472} = 2.77'' \quad \text{Use } 2.5''$$

4' - 0" from support

$$V_u = 144,365.84 - \left[1.4(676.58 + 268.55) + 1.7 \times 1,038\right] \times 4$$

$$= 132,014.71 \text{ lb.}$$

Shear force carried by concrete

$$V_u' = 107.5 \times 6 \times 26 = 16,770.0 \text{ lb}$$

Shear force carried by inclined main bars

$$V_u'' = 95,472.00 \text{ lb.}$$

Use #3 stirrups, the spacing

$$S = \frac{0.85 \times 0.22 \times 40000 \times 26}{132,014.71 - 167.70 - 95,472} = 9.84" \text{ use } 9"$$

Using these two spacings 2.5" and 9" the total number of stirrups

$$N = \frac{12}{2.5} + \frac{18 \times 12}{9} = 5 + 24 = 29 < 39$$

Thus horizontal shear governs design. The arrangement of stirrups is shown in Fig. 4.17

Checking distribution of flexural reinforcement

$$d_c = 2"$$

The average effective tension area of concrete

$$A = \left(2 + \frac{19}{8} + \frac{19}{8} + 2\right) \times 6/6 = 8.75 \text{ in}^2$$

Service load moment

M_d = 650.24 ft-kip $\quad$ M_l = 775.13 ft-kip

$$k = \sqrt{(8 \times \frac{9.36}{6x58})^2 + 2 \times 8 \times \frac{9.36}{6x58}} - 8 \times \frac{9.36}{6x58} = 0.48$$

$$Z = 37.51 \sqrt[3]{2 \times 8.75} = 97.38 < 175$$

O.K.

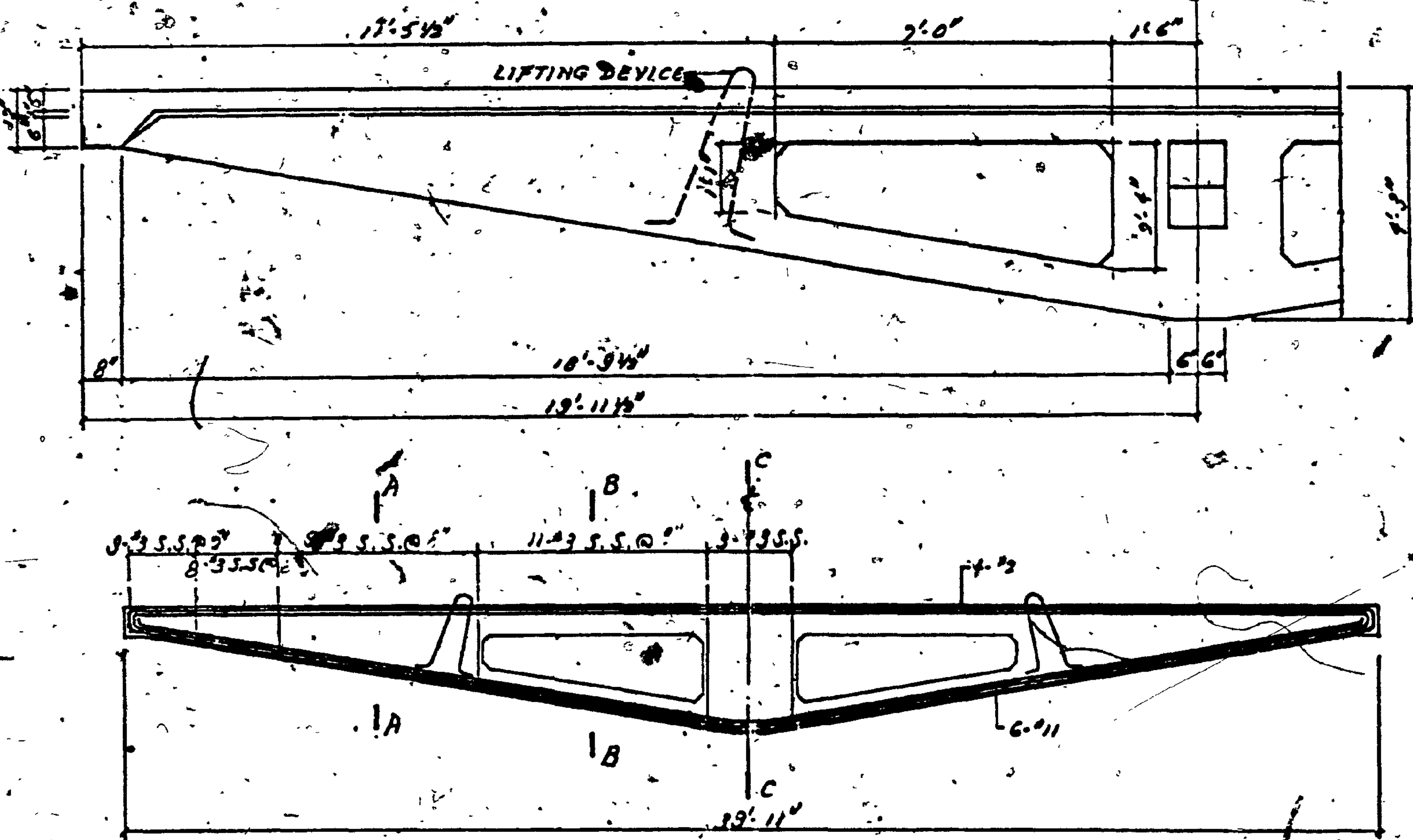


Fig. 4.17 Elevations of girder

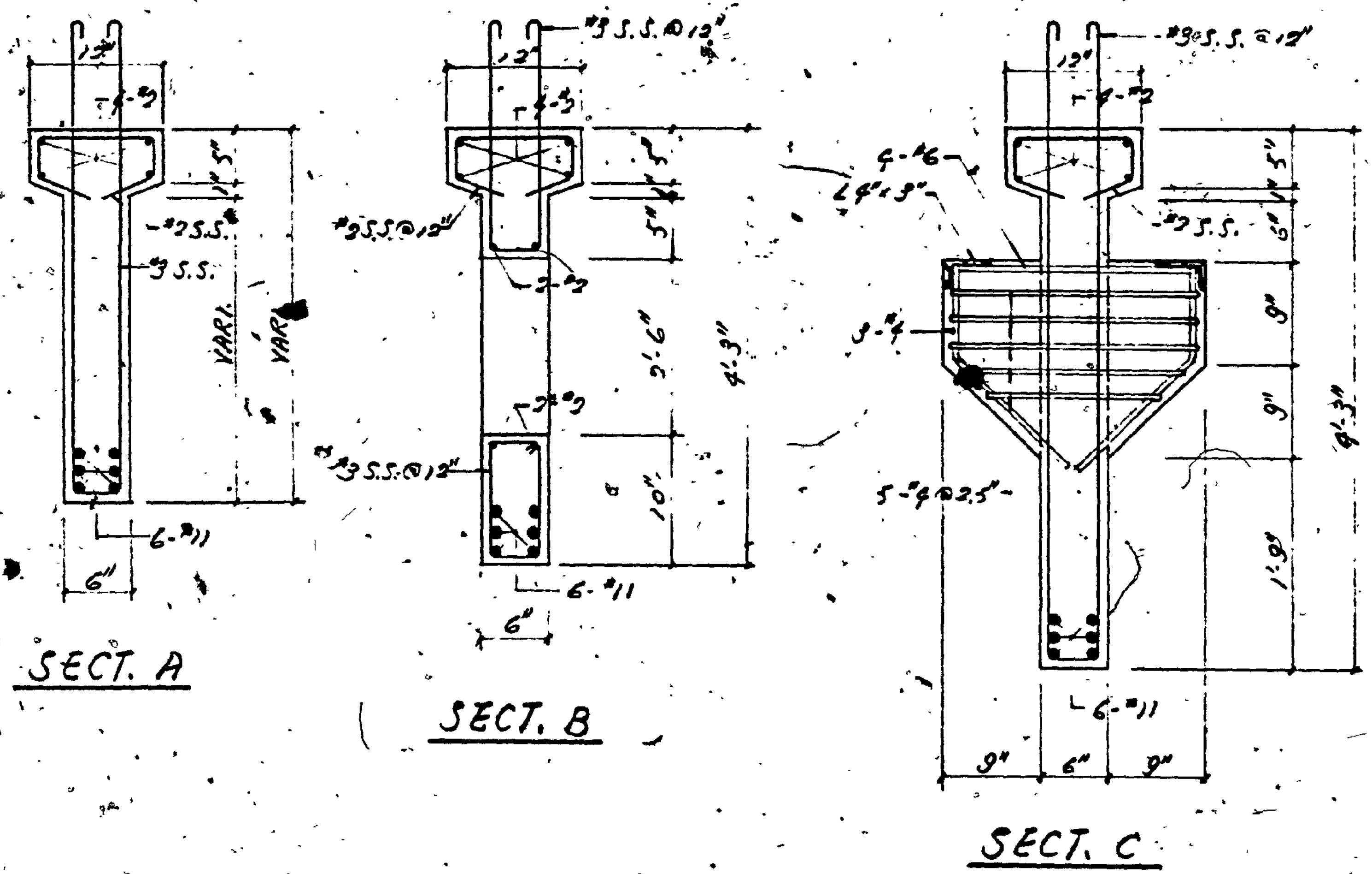


Fig. 4.18 Details of reinforcement of girder

4.5 BRACKET

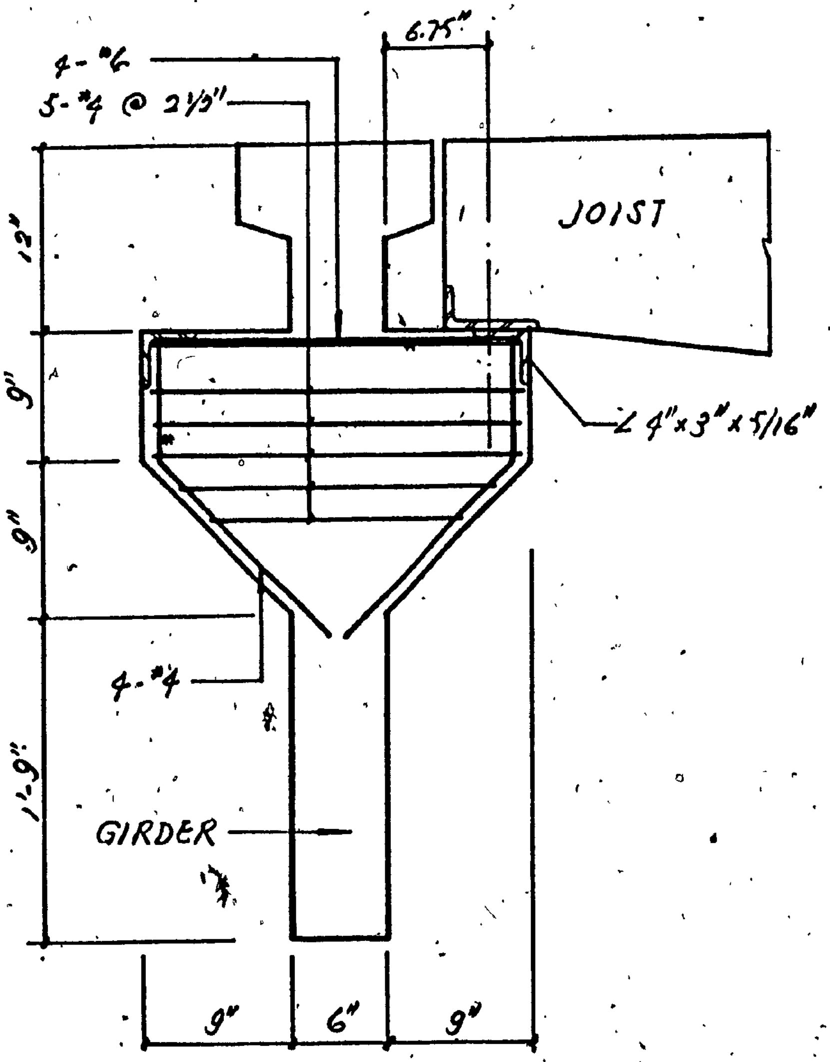


Fig. 4.19

Vertical load

V_u = 97902.05 lb.

Width of bracket

b = 12"

Shear span L_v = 6.75"

ACI - 10.14 gives ultimate bearing capacity as 0.85 f_c'

Using $\phi = 0.70$ (ACI Code 9.2 - 1.4)

Shearing force

$$V_u = 0.85 \; f_c' \phi bd$$

The width of steel bearing plate

$$d_b = \frac{V_u}{0.85 \; f_c' \; \phi \; b} = \frac{97902.05}{0.85 \times 4000 \times 0.7 \times 12}$$

$$= 3.43''$$

use $\angle$ 4" x 3" x 5/16" , 12" long

Assume $N_u/V_u = 0.2$

The shear stress is given

$V_u = 0.85 \times 800 = 680$ psi

The minimum depth of bracket is

$$d = \frac{V_u}{bV_u} = \frac{97902.05}{12 \times 680} = 12''$$

Try d= 18" (depth of bracket).

$L_{v/d} = 6.75/18 = 0.38 < 0.5$

From ACI Design Handbook, (Shear 15)

$K_{v1} = 0.014$, $K_{v2} = 1.0$ and $K_{v3} = 0.0196$

Shear friction reinforcement is obtained

$$A_{vf} = K_{v1}K_{v2}V_u = 0.014 \times 1 \times 97{,}902.05/1{,}000 = 1.37 \text{ in}^2$$

$$A_{sh} = K_{v3}N_u = 0.0196 \times 0.2 \times 97{,}902.05/1000 = 0.38 \text{ in}^2$$

$$\text{Total} \quad A_s = 1.75 \text{ in}^2$$

Checking the maximum and minimum reinforcement ratio:

$$\rho_{max} = 0.13\ f_c'/f_y = 0.13 \times 4{,}000/60{,}000 = 0.0087$$

$$\rho_{min} = 0.04\ f_c'/f_y = 0.04 \times 4{,}000/60{,}000 = 0.0027$$

$$A_{s(max)} = 0.0087 \times 12 \times 18 = 1.88 > 1.75$$

$$A_{s(min)} = 0.0027 \times 12 \times 18 = 0.58 < 1.75 \qquad \text{O.K.}$$

Use 4 - #6 (1.77 > 1.75)

$$A_h = \tfrac{1}{2} A_s = 0.5 \times 1.75 = 0.875 \text{ in}^2$$

Use 5 - #4 (1.00 > 0.875)

4.6 SHEAR FRICTION AT COLUMN HEAD

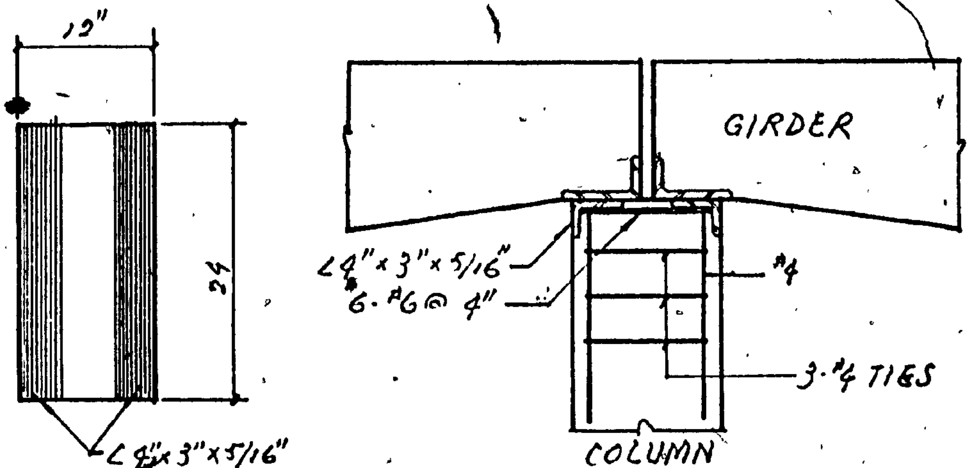


Fig. 4.20

Vertical load carried by girder.

$$V_u = 144365.84 \text{ lb.}$$

The width of steel bearing,

$$d_p = \frac{144365.84}{0.85x0.7x5000x12} = 4.0'' \quad \text{Use } \angle 4"x3"x5/16"$$

From ACI design handbook (Shear 15.)

$$K_{v1} = 0.014, \quad K_{v2} = 1 \text{ and } K_{v3} = 0.0196$$

Shear friction reinforcement is

$$A_{vf} = 0.014 \times 1 \times 144365.84.1000 = 2.02 \text{ in}^2$$

$$A_{sh} = 0.0196x0.2x144365.84/1000 = 0.57 \text{ in}^2$$

$$\text{Total} \qquad A_s = 2.59 \text{ in}^2$$

Use 6 - #5 @4" (2.64 > 2.59)

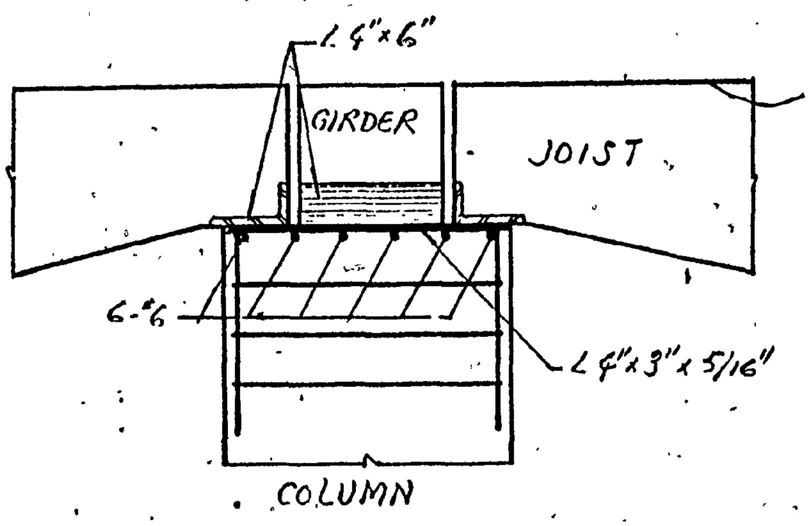


Fig. 4.21

Vertical load carried by joist at each end

$$V_u = 97,902.05 \text{ lb}$$

Bearing capacity

$$V_u = \frac{97,902.05}{0.7x8x5} = 3,496.5 \text{ psi}$$

Allowable bearing stress

$$V_u = 0.85\ f'_c = 0.85 \times 5,000 = 4,250 > 3,496.5 \quad \text{O.K.}$$

For shear friction the reinforcement is

$$A_s = 1.75 \text{ in}^2 \text{ (see bracket)}$$

The cross sectional area of ∠ 4" x 3" x 5/16" is

$$A_s = 2.09 > 1.75 \quad \text{O.K.}$$

JOIST AND GIRDER

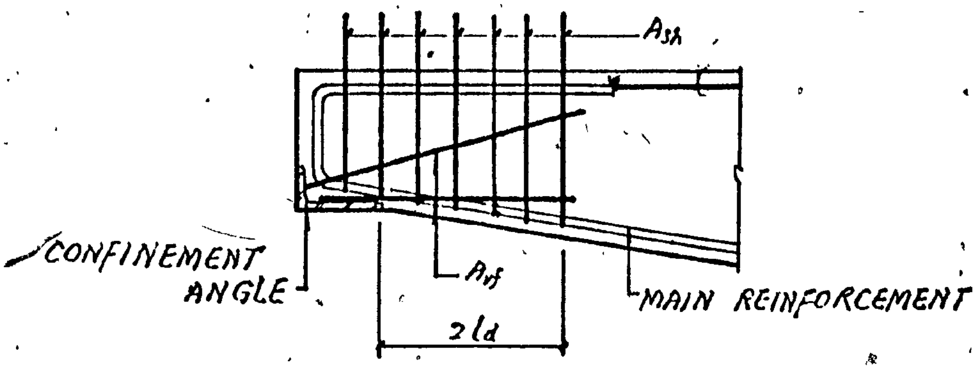


Fig. 4.22

For girder

$$V_u = 144365.84 \text{ lb}$$

$$N_u = 0.2 \times 144365.84 = 28873.17 \text{ lb}$$

$$A_{vf} = \frac{1}{\phi f_y}\left(\frac{V_u}{\mu} + N_u\right)$$

where $\phi = 0.85$. $f_y = 60000$ psi, $\mu = 1.4$

$$A_{vf} = \frac{1}{0.85 \times 60000}\left(\frac{144365.84}{1.4} + 28873.17\right) = 2.59 \text{ in}^2$$

Use 6 - #6 (2.64 > 2.59)

$$2l_d = 0.04\ A_{so}\ f_y/\sqrt{f'_c} = 0.04 \times 0.44 \times 60000/\sqrt{4000}$$

$$= 16.7" \qquad \text{Use } 18"$$

$$A_{sh} = \frac{A_{vf}}{\mu} = \frac{2.64}{1.4} = 1.89 \text{ in}^2$$

Use #3 stirrups $A_v = 0.22$

$$N = \frac{1.89}{0.22} = 8.57 \quad \text{Use 9 nos @ 2"}$$

For joist

$$V_u = 97,902.05 \text{ lb.}$$

$$N_u = 0.2 \times 97,902.05 = 19,580.41 \text{ lb.}$$

$$A_{vf} = \frac{1}{0.85 \times 60000}\left(\frac{97,902.05}{1.4} + 19,580.41\right)$$

$$= 1.76 \text{ in}^2$$

Use 6 - #5 (1.86 > 1.76)

$$2l_d = 0.04 \times 0.31 \times 60000/\sqrt{4000} = 11.76" \quad \text{Use 12"}$$

$$A_{sh} = \frac{1.86}{1.4} = 1.33 \text{ in}^2$$

Use #3 stirrups $A_v = 0.22$

$$N = \frac{1.33}{0.22} = 6.04 \quad \text{Use 6 nos @ 2"}$$

CONCLUSION

Precast concrete elements of large spans are very often employed in various projects. The panel size used in this application example is 40' by 40' which is the prevaling size in Europe and America.

This example is designed to satisfy the identified requirements and provisions stated in the ACI Code. The properties of materials, the designed methods of the elements and the safety of the roof structure are considered in this application.

Economy is achieved by employing thin slab panels, and openweb trapezoidal joists and girders. Materials used in this design are:

Concrete - 45 lb/sq. ft.

Steel - 3.95 lb/sq. ft.

All the precast elements are easy to prefabricate, transport and erect. A roof structure with this standard panel size may be designed to meet a variety of loading conditions.

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